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DAMAGE TO STRUCTURES BY FRAGMENTS AND BLAST

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Ballistic Research Laboratories
Aberdeen Proving Ground
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ABSTRACT

This report presents a broad treatment of fragment-blast damage, starting with the mechanisms of failure and then presenting methodology for computing damage from fragment-blast weapons. Calculations are based on an assumed mode of failure after fragment damage takes place. An example problem is given assuming a buckling type failure for both wing and fuselage. The report also presents failure calculations for cylindrical shells and compares the results obtained for collapse with those for collapse-hinge buckling and lobar buckling.

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I. Physical processes in blast, perforation and penetration

For weapons having blast characteristics alone, the loading usually engulfs the structure and can produce lethal damage through overall deformation of the vehicle or through damaging of a critical section. For fragment type weapons the high velocity fragments perforate parts of the structure and can produce lethal damage if critical portions of the structure are blown out by the fragments. There are other weapons which have both fragment and blast characteristics and these can produce structural damage by the fragments first taking out part of the structure or plastically deforming the structure (without perforation) and then the blast finishing the damage.

A description of the processes involved in penetration and perforation of very high velocity fragments against plate type targets is contained in an earlier reference.[1*] When a high speed fragment or projectile reaches the surface of a target several things can occur depending upon the characteristics of the fragment and target. The fragment can perforate the target completely and continue traveling with a small change in velocity due to the resistance of the target. If the target is an aircraft or missile structure the fragment can enter on one side and, if the speed is high enough, and the target resistance low enough, it will leave on the other side. This will happen in very thin structures which offer little resistance to perforation such as shown in Figure 1. For thicker targets offering more resistance, the fragment could perforate the entrance side and experience a substantial decrease in velocity -- thus having its kinetic energy considerably lowered. Part of this energy change will go into perforating and deforming the target and projectile and part will go into heating the fragment and target possibly to the point of vaporization. In the meantime, as perforation occurs, liquid spall fragments can be formed on the inside surface. This physical process is illustrated in Figure 2. In some cases the entering fragment will not be vaporized because the initial velocity is not large enough. In this case the spall fragments together with the original broken up fragment will load the exit side as shown in Figure 3. If the velocity of the original fragment is not high enough for perforation then penetration could occur as shown in Figure 4. If the velocity is not sufficient for penetration, then the fragment might cause plastic deformation of the structure and bounce off such as shown in Figure 5.

In the process illustrated in Figure 2 the shock will undoubtedly blow out the entrance side of the structure since the energy of the entering fragment has been converted to heat during the perforation process and the fragment will be converted mostly to liquid and gas at the rear of the entrance side. In the process shown in Figure 3, the spall fragments will probably blow out the exit side unless their energy is absorbed in internal components. In this case fragments will be formed at the inside of the entrance panel and be projected toward the exit

*Superscripts refer to references listed at the end of the report.

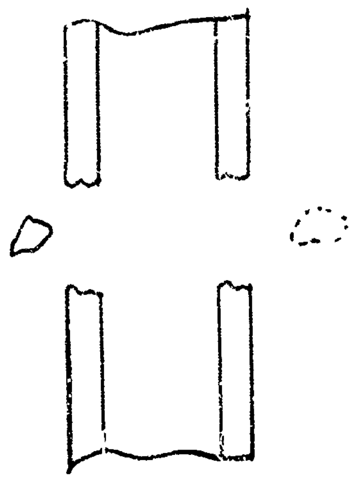


Fig. 1 Perforation

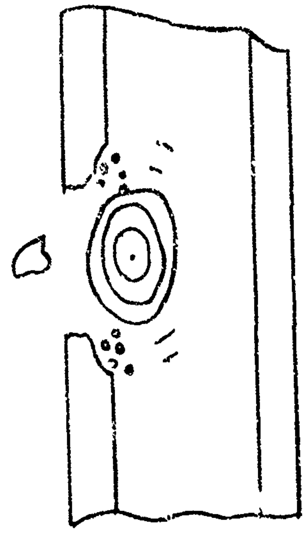


Fig. 2 Melting and Vaporization

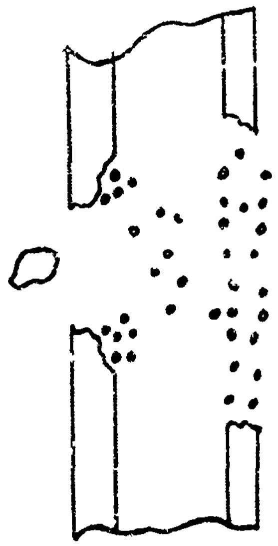


Fig. 3 Breakup

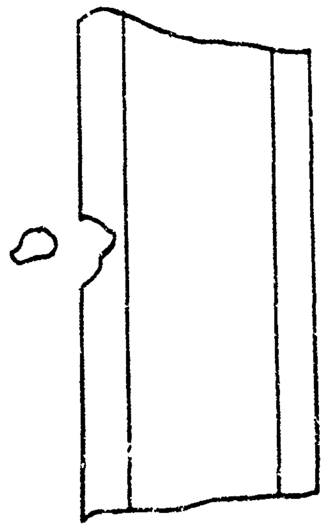


Fig. 4 Penetration

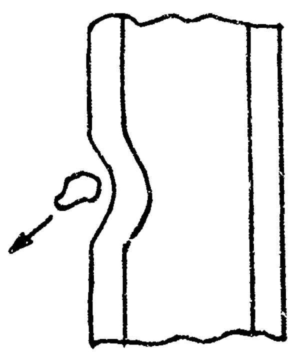


Fig. 5 Plastic Deformation and Bounce Off

side. Liquification and vaporization will ordinarily occur at velocities of impact greater than 25,000 ft./sec. for steel fragments perforating an aluminum target. For any velocities lower than this, the phenomena described in Figures 2, 3, 4 or 5 will probably occur.

II. Energy concepts as a general description of damage

In blast work one of the most common ways to show damage results is on an isodamage curve. This isodamage curve consists of a pressure-impulse plot such as shown in Figure 6

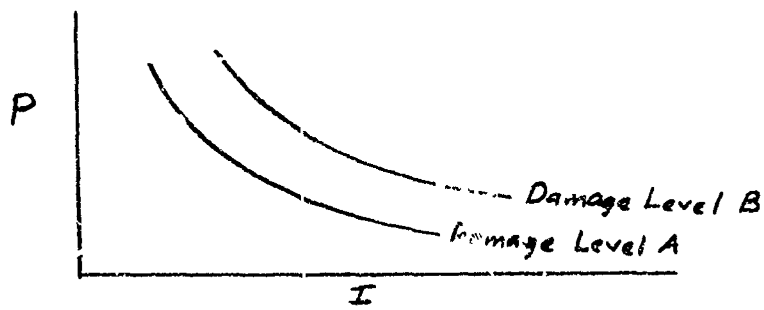


Fig. 6 Isodamage Blast Curve

It can be shown theoretically[2] that the curves of constant damage plotted in this P-I plane are approximately constant blast energy curves since the blast energy flux (energy per unit area) can be written

$$E_B = \frac{PI}{2\rho_0 C_0} \qquad [1]$$

where E_B = energy flux in the explosion at the target

I = Impulse per unit area in the blast

P = blast overpressure at the target

ρ_0 = air density

C_0 = sound velocity in the air

If the energy available to do damage (i.e. energy flux) is equated to the energy absorbed in the structure then the curve plotted in the P-I plane is both a constant damage curve as well as a curve of constant available blast energy.

If we now consider a weapon in which fragments are a mechanism of damage, the available energy from each fragment is

$$E = \frac{1}{2} M v^2 \qquad [2]$$

where M = mass of fragment

v = velocity of fragment at target

A curve plotted in the v - M plane will look like the curve shown in Figure 7

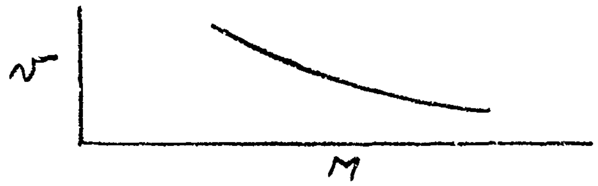


Fig. 7 Isodamage Fragment Curve

By the same reasoning used for blast, the curves plotted in the v - M plane for fragments are also constant energy curves and the isodamage curve for fragments is the v - M curve.

III. Basic Equations for Fragment Perforation and Collapse of Cylindrical Structures

A. Fragmentation -Perforation

If the mass and velocity of the incoming fragments are of sufficient magnitude then these fragments will perforate the structure. The Thor equation for determining the change in velocity for fragments which perforate a target is as follows:

$$V_r = V_s - 10^c (eA)^\alpha m_s^\beta (\sec \theta)^\gamma V_s^\lambda \qquad [3]$$

where

V_r is the fragment residual velocity in fps after perforation has occurred

V_s is the fragment striking velocity in fps

e is the target thickness in inches

A is the average impact area of the fragment in square inches

m_s is the weight of the original fragment in grains

θ is the angle between the trajectory of the fragment and the normal to the target material

$c, \alpha, \beta, \gamma, \lambda$ are constants determined for each material and are shown in Table 1:

Table 1 Perforation

Residual Velocity Constants
(Steel Fragments)

Material	c	α	$-\beta$	γ	$-\lambda$
Magnesium	6.9	1.1	1.2	1.1	.09
Aluminum Alloy	7.0	1.0	1.1	1.2	.14
Titanium Alloy	6.3	1.1	1.1	1.4	.17
Cast Iron	4.8	1.0	1.1	1.0	.52
Face Hardened Steel	4.4	.7	.8	1.0	.43
Mild Steel	6.4	.9	.9	1.3	.02
Hard Steel	6.5	.9	.9	1.3	.02
Copper	2.8	.7	.7	.8	.80
Lead	2.0	.5	.5	.7	.82
Tuballoy	2.5	.6	.6	.9	.83

For fragments which are on the threshold of perforation (i.e. if the fragment had a lower velocity, it would not perforate) the velocity is given by

$$V_o = 10^{c_1} (eA)^{\alpha_1} m_s^{\beta_1} (\sec\theta)^{\gamma_1} \qquad [4]$$

where the constants are given in Table 2.

Table 2 Perforation
Threshhold Velocity Constants
(Steel Fragments)

Material	c_1	α_1	$-\beta_1$	γ_1
Magnesium	6.4	1.0	1.1	1.0
Aluminum Alloy	6.2	.9	.9	1.1
Titanium Alloy	7.6	1.3	1.3	1.6
Cast Iron	10.2	2.2	2.2	2.2
Face Hardened Steel	7.7	1.2	1.4	1.7
Mild Steel	6.5	.9	1.0	1.3
Hard Steel	6.6	.9	1.0	1.3
Copper	14.1	3.5	3.7	4.3
Lead	11.0	2.7	2.7	3.6
Tuballoy	14.8	3.4	3.5	5.0

A fragment with a velocity slightly greater than this velocity (eq. [4]) will knock out a section of the structure of thickness e and area A. For a given fragment distribution in which we have effective area A_1 over which fragments perforate the weapon will knock out this area of the structure.

Equations [3] or [4] are used only in one way. Given the mass of the fragment m_s, its projected area A, the target thickness e, the angle θ, and the appropriate constants, this equation gives the minimum velocity

of fragment which will cause perforation. Let the velocity of the fragment be V. If $V < V_o$ no perforation will occur, if $V > V_o$ perforation will occur and the fragmentation damage will be done. If the mass and velocity parameters of the warhead and the thickness of the target are of appropriate value so as to cause perforation over a large enough area, then perforation could be sufficient to kill the structure alone without any further considerations.

B. Fragments - No perforation

If fragments do not perforate they still could inflict some damage on the structure by imparting their energy to the structure and causing some plastic deformation by penetrating the skin and/or hitting the structure and bouncing off. Let the kinetic energy of a fragment (which either penetrates or bounces off) of mass m_s and velocity v_s be denoted by E_s, then

$$E_s = \frac{1}{2} m_s v_s^2 \qquad [5]$$

Assuming no energy losses, the total energy imparted by these fragments will be the sum over s, i.e.*

$$E_A = \sum_s \frac{1}{2} m_s v_s^2 \qquad [6]$$

C. Collapse in a cylindrical structure - Missile bodies and fuselages

1. Collapse due to non perforating fragments

It is conceivable that the total energy from non perforating fragments could be sufficient to cause collapse in a cylindrical shell structure such as shown in Fig. 8 and 9 ** Assuming a linear hardening material such as shown in Figure10, the energy absorbed in the collapse mode can be written:[4]

$$\bar{V} = \frac{\sigma_s h a l}{\sqrt{3}} \left\{ \frac{\sqrt{3}(1-\lambda)}{2 e_s (1-\nu^2)} \bar{I}_1 + \lambda \bar{V}_1 - \lambda \sqrt{3}\, \pi e_s \right\} \qquad [7]$$

where σ_s = yield stress of material in pure tension
h = thickness of material
l = length of shell
λ = hardening parameter
e_s = yield strain
ν = Poisson's ratio

Temperature effects are considered by adjusting σ_s and λ. The values of $\bar{I}_1$, $\bar{V}_1$ are given in Fig.11 The above relation, [7] is the energy necessary to cause collapse with a given deflection as shown in Figures 9 and 11

*In order to assume that the energy from the fragments is imparted to the structure when there is no perforation, the value of v_s must approach V_o (the perforation velocity); otherwise the fragments will just bounce off and only impart a small portion of their energy thereby causing only elastic deformation in the structure.

**Since we are primarily concerned with focused fragment - blast warheads in this report, the nature of the loading will eliminate the lobe buckling type of failure which is characteristic of blast load engulfment.

2. Collapse with non perforating fragments together with blast

It is also possible that a combination of non perforating fragments and blast could cause considerable damage even if no perforation occurs. This is accomplished by energy transfer. First the fragments impart their energy E_{A} and deform the shell plastically as described above. The blast then comes along and imparts its energy E_B. For a given overpressure P_s the value of $\frac{E_f \rho_o C_o}{T P_s^2}$ is obtained from Figure 12. E_f is the energy per unit area in the blast, ρ_o is the air density, C_o is the sound velocity in the ambient air, T is the positive duration of the overpressure, thus

$$E_B = E_f A \qquad [8]$$

where A is the area of the target facing the blast. The total energy imparted to the target by non perforating fragments plus the blast is

$$E_{total} = E_A + E_B \qquad [9]$$

It is this energy (E_{total}) which is compared with $\bar{V}$ to determine what degree of collapse (i.e. what w_o) the combined weapon will inflict on the target.

3. Collapse with perforating fragments and blast

If the fragments have already perforated then they have knocked out a given area of the shell. This means that the rest of the shell is only capable of absorbing energy over its remaining area. The energy necessary to inflict collapse damage with deflection w_o on the perforated structure is

$$\bar{V}_p = \bar{V} \times \frac{\text{Remaining area of the body after Penetration takes place}}{\text{Original area of the body}} \qquad [10]$$

We compare $\bar{V}_p$ with E_B (the energy available from the blast) to see how much more damage will occur in the perforated shell.

4. The combined case - see Figure 13

For most cases some of the fragments will perforate thus weakening the structure, the blast will then come along and further deform the structure and then the slower fragments will either penetrate or bounce off and plastically deform the structure. The energy available from the weapon to damage the perforated structure is

$$E_A = E_B + (E_{frags})_{non\ perforating}$$

This is the energy to be compared with V_p in order to determine how much damage this available energy could inflict on the perforated structure.

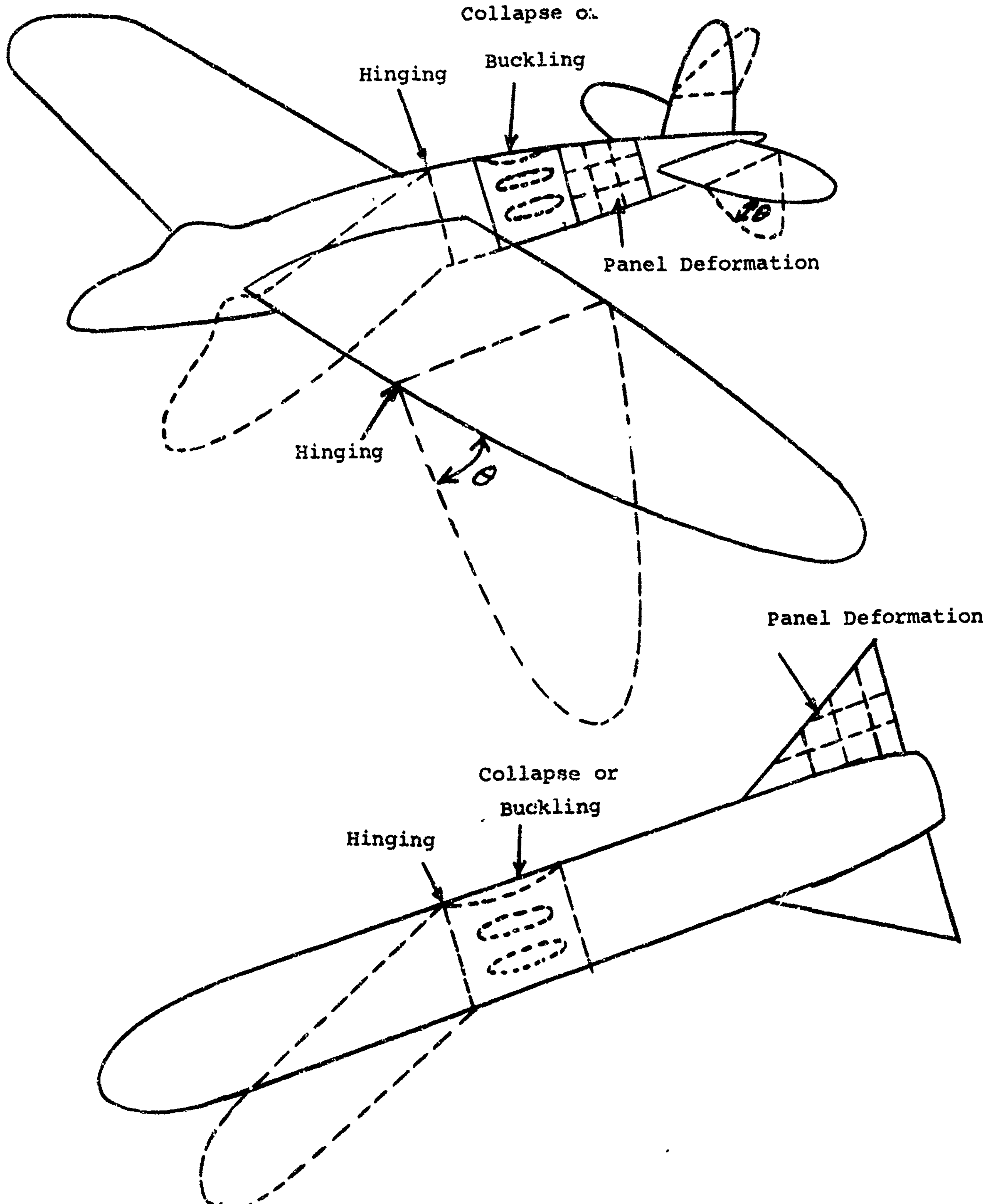


Fig. 8 Schematic of Various Types of Failure

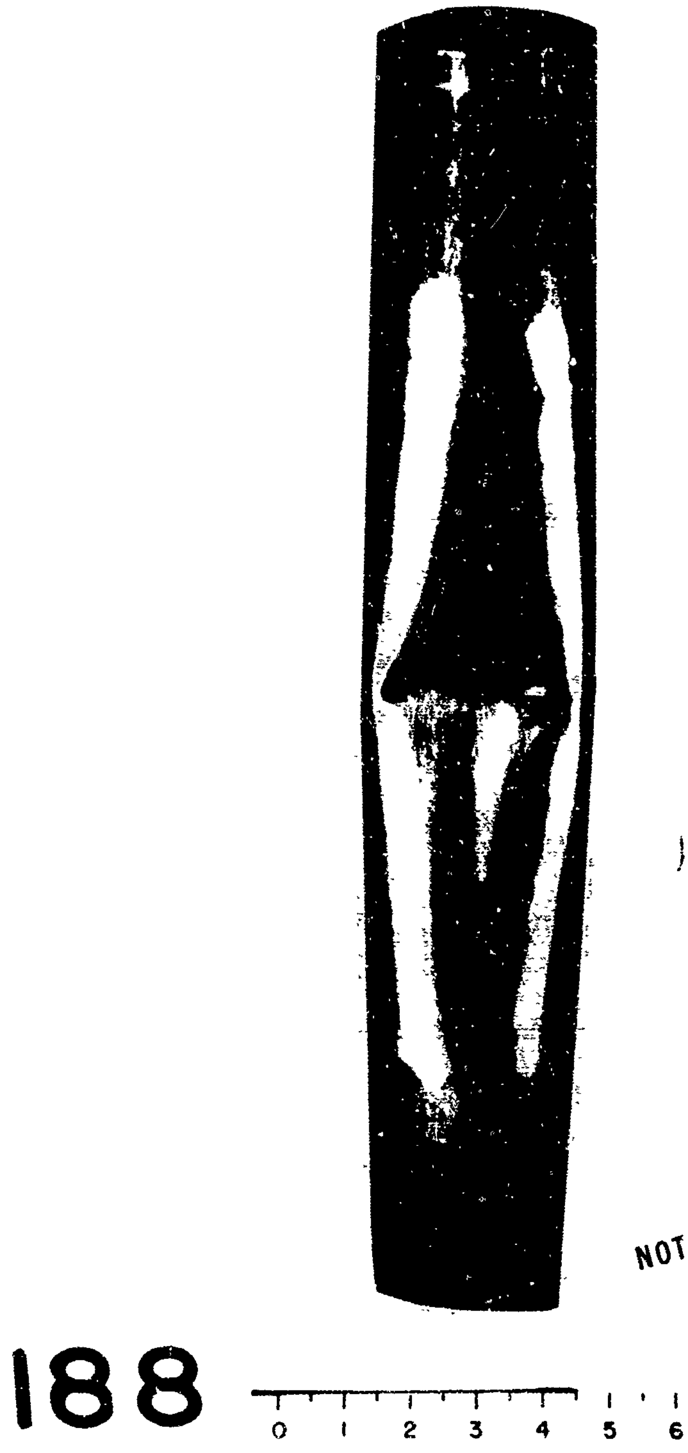


Fig. 9 · Collapse Pattern

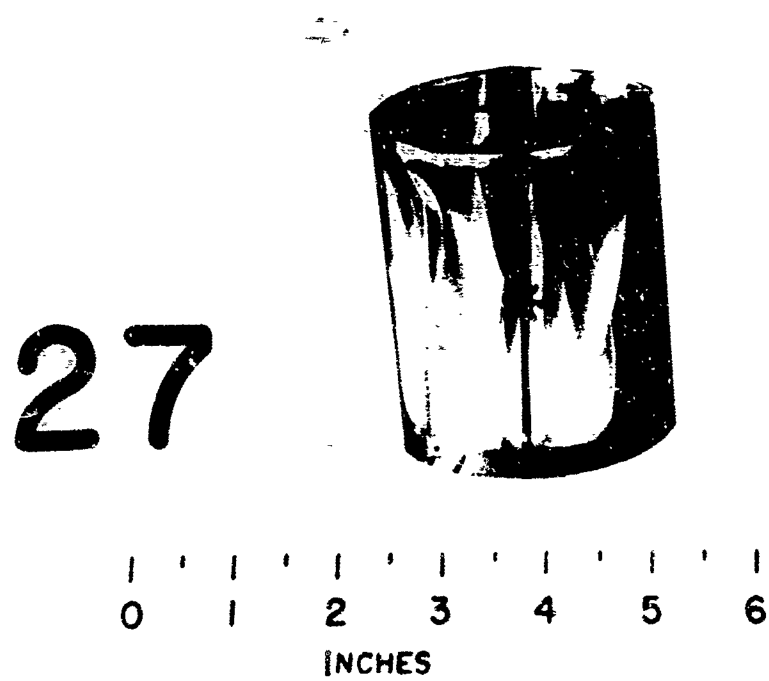
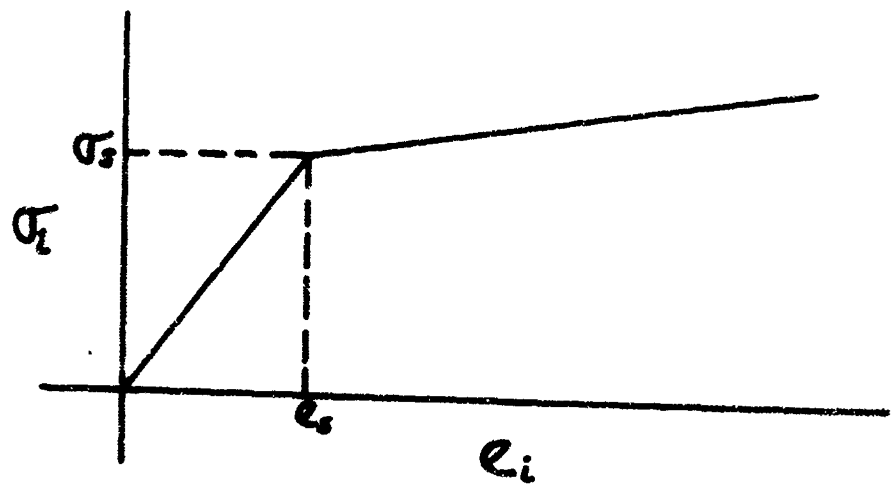


Fig. 9a Buckling Pattern



$$\frac{\sigma_i}{\epsilon_i} = E\left[1 - \omega(\epsilon_i)\right]$$

$$\omega(\epsilon_i) = 0 \quad \text{for} \quad \epsilon_i < \epsilon_s$$

$$\omega(\epsilon_i) = \lambda\left(1 - \epsilon_s/\epsilon_i\right) \quad \text{for} \quad \epsilon_i > \epsilon_s$$

$$\lambda = 1 - \frac{1}{E}\frac{d\sigma_i}{d\epsilon_i}$$

Figure 10 Elastic-Linear Hardening Law

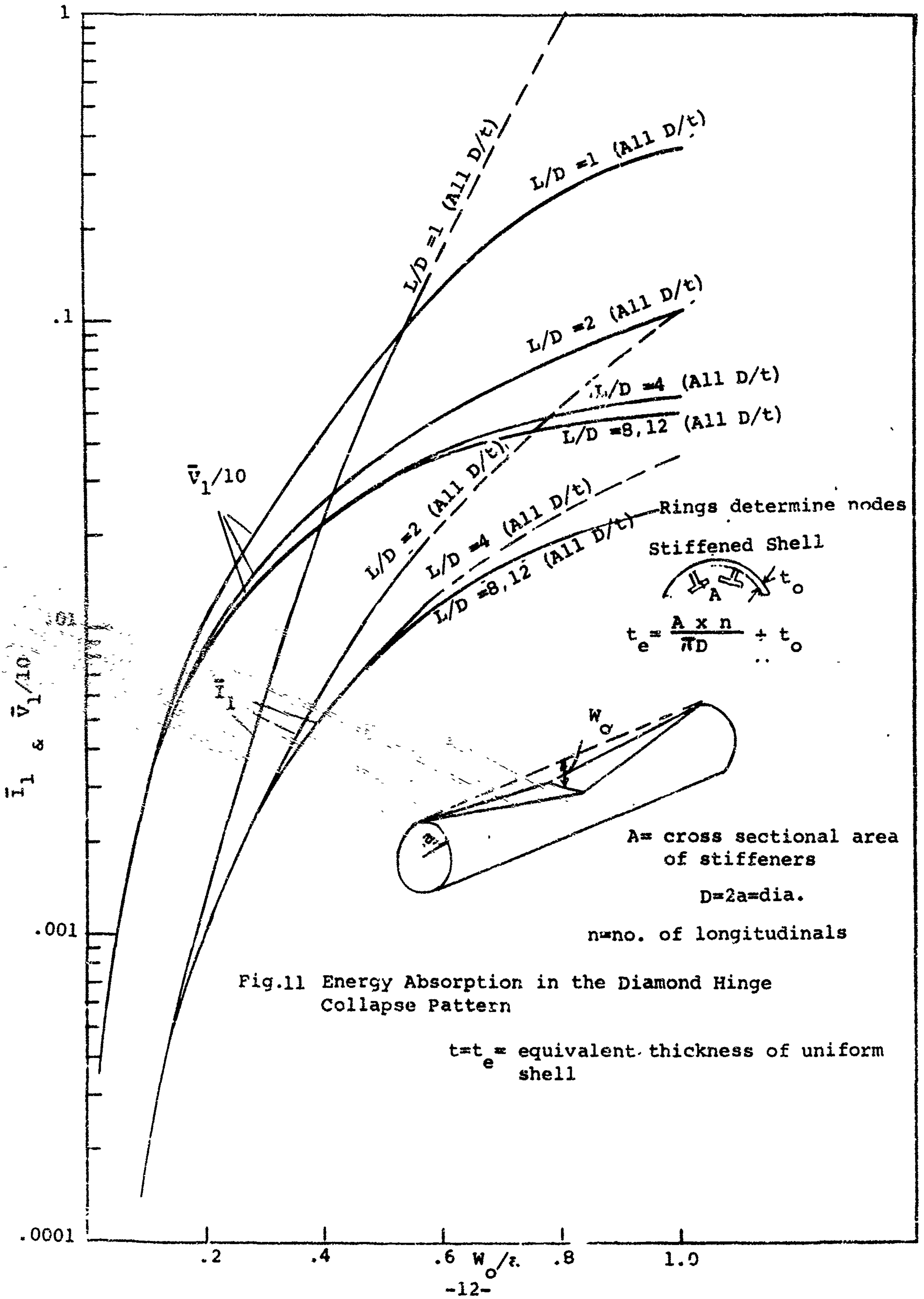


Fig.11 Energy Absorption in the Diamond Hinge Collapse Pattern

$t=t_e$ = equivalent thickness of uniform shell

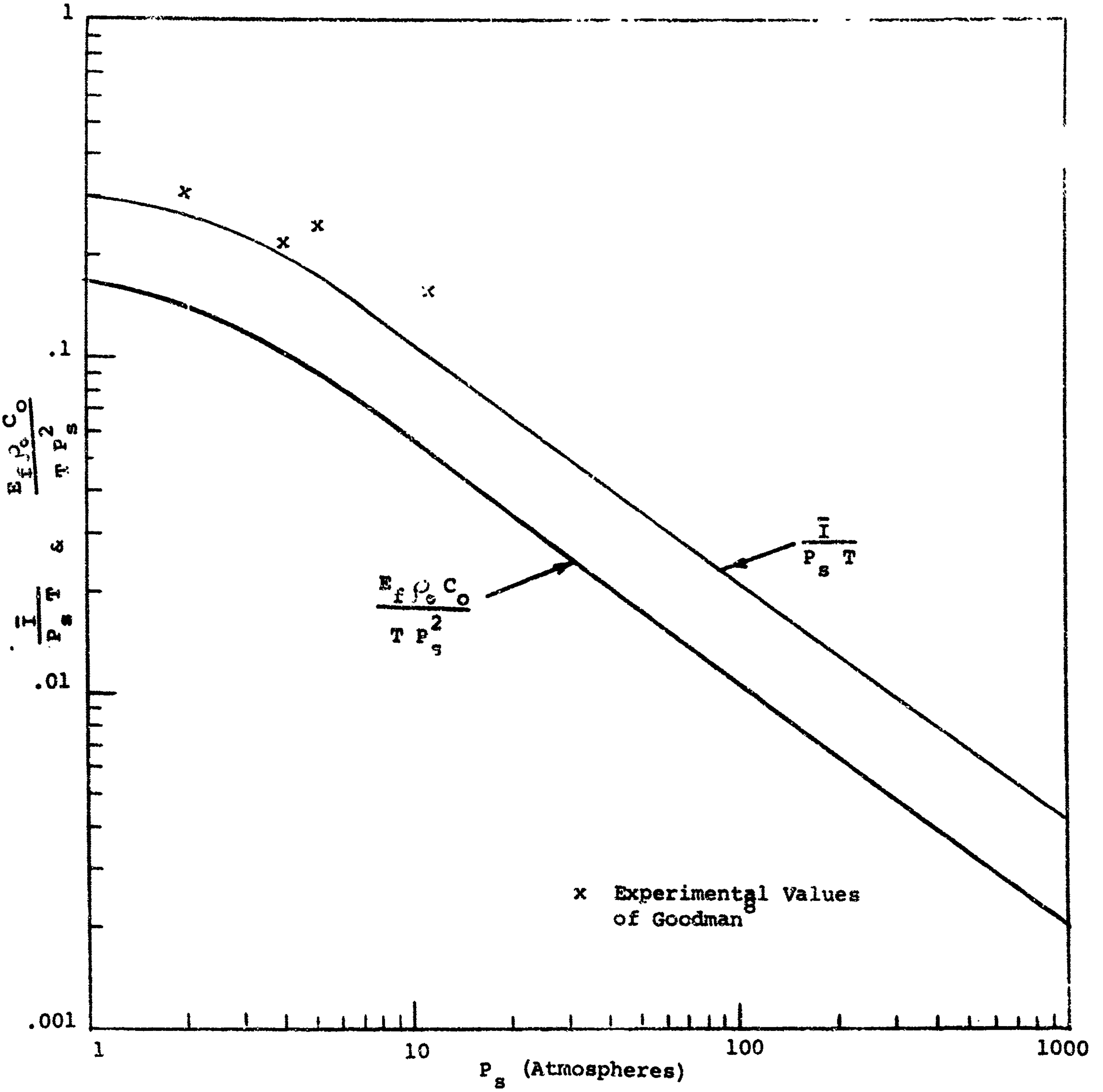


Fig. 12 Nondimensional Impulse and Energy Values

After Perforation

Residual Energy Available from Weapon
(non-perforating frags +blast)

= Energy Absorbed in Weakened (Perforated) Section

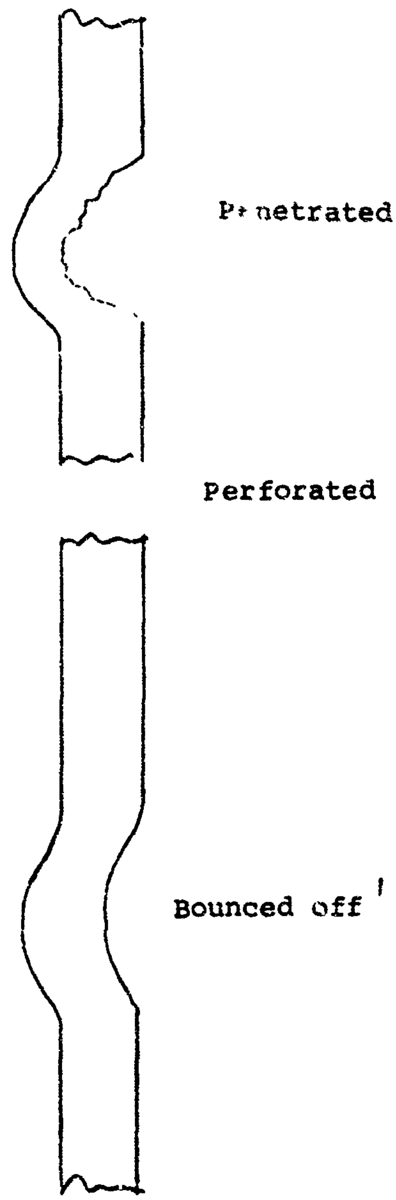


Fig. 13 Combined Damage

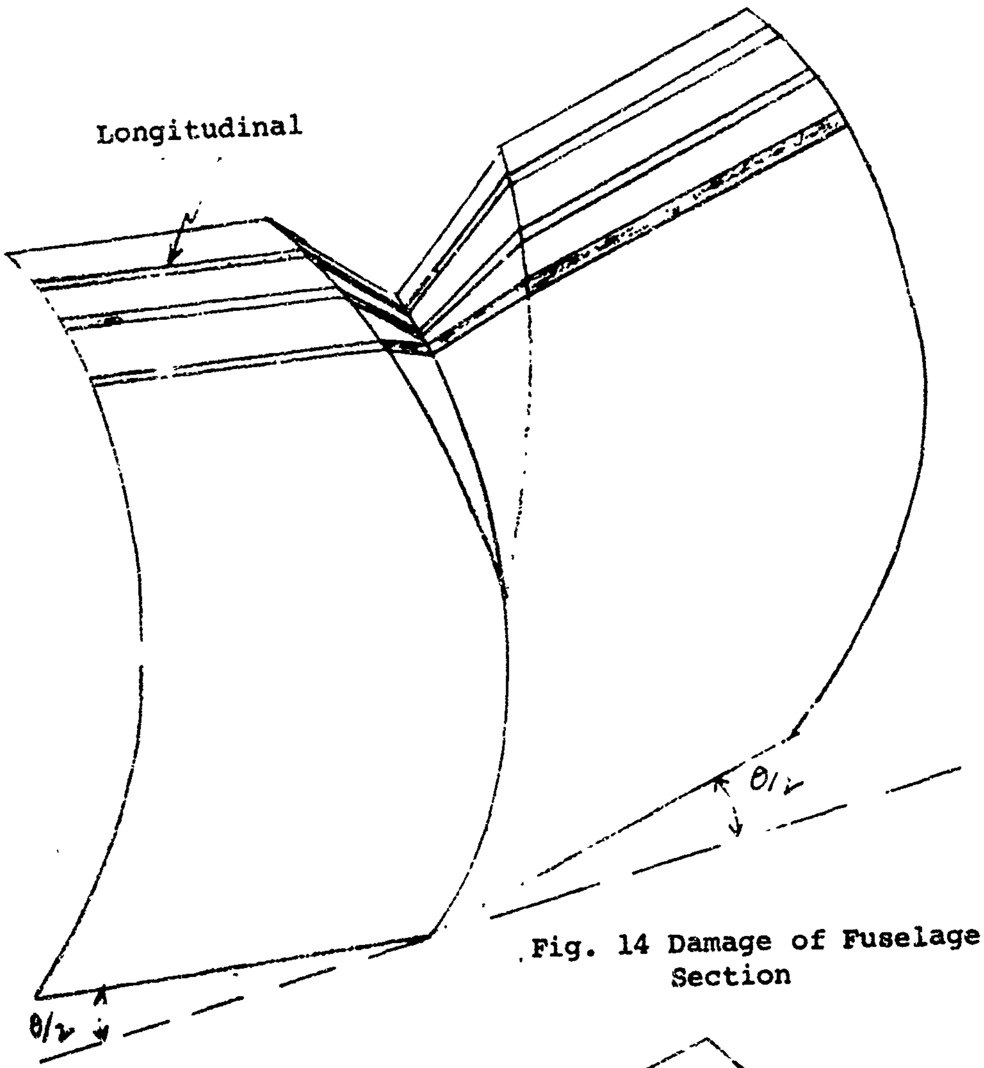


Fig. 14 Damage of Fuselage Section

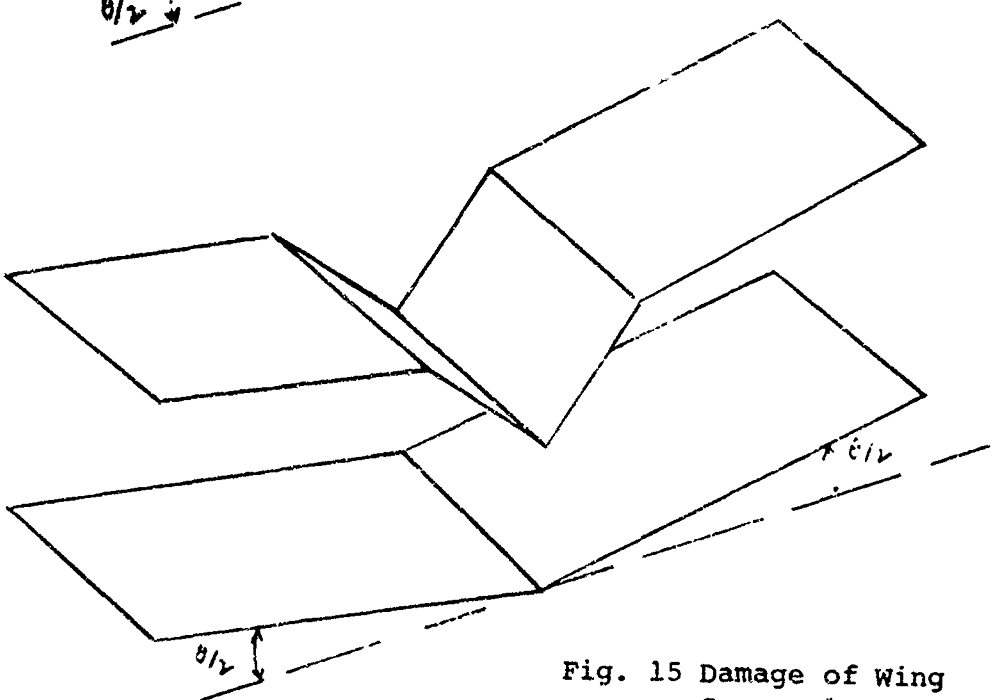


Fig. 15 Damage of Wing Cover Plates

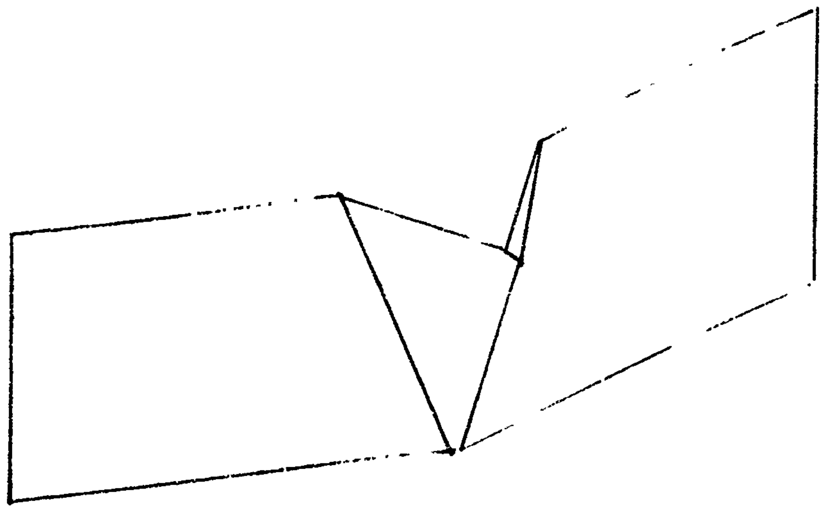


Fig. 16 Damage of Stiffener Web

IV. Buckling Failure of Wings and Fuselages

It is assumed that failure both in a wing (or other control surface) and in a fuselage takes place by local buckling of the skin, stringer, and spars such as shown in Figures 14 - 16 The section at which failure will occur will be the section which is weakened by fragment perforation. In order to determine whether steel fragments will perforate aluminum structures we merely have to apply the THOR equations. After perforation takes place the local section will be weakened because it has less load carrying area. This remaining weakened structure will then be exposed to the oncoming blast.

The structural theory of post failure behavior of buckled structures was worked out by D'Amato[5] some years ago. Neglecting any elastic vibrational energy, the energy absorbed by the structure is the elastic energy in taking the structure up to failure at the weakened critical section plus the plastic energy in rotating the structure through angle θ (see Figure17) around the critical section.

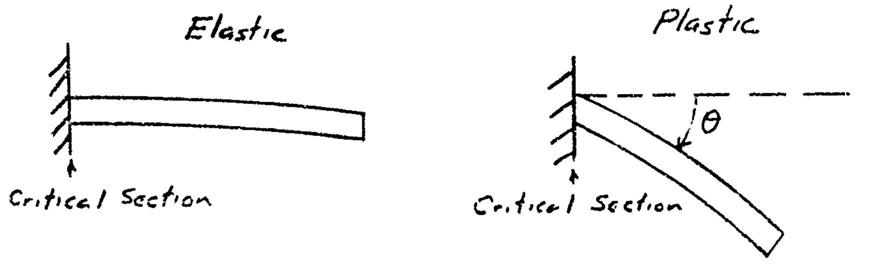


Fig. 17 Configurations of the Structure

At the critical section there is a resisting moment which is schematically shown in Figure 18

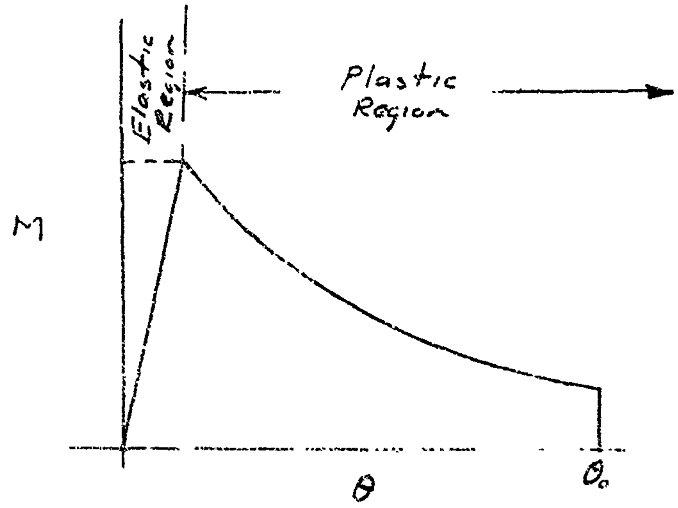


Figure 18 Hinge Resisting Moment

The area under the M - θ curve is the strain energy absorbed as the structure undergoes failure.

Considering the structure as a single degree of freedom deforming in the hinge mode (see Fig. 4), the equation of motion is as follows:

$$I\ddot{\theta} = M(t) - M(\theta) \qquad [11]$$

where θ = rotation angle

I = mass moment of inertia of the hinged section around an axis through the critical section

$M(t)$ = external moment applied to the weakened (perforated) structure by the blast

$M(\theta)$ = resisting moment at the critical section

Following the logic of Ref. 2, the external work done by loading up to time t is

$$W(t) = \int_0^{\theta(t)} M(t)\, d\theta = \int_0^t M(t) \frac{d\theta}{dt} dt \qquad [12]$$

from [11]

$$\frac{d\theta}{dt} = \frac{1}{I} \int_0^t [M(t) - M(\theta)]\, dt \qquad [13]$$

Thus

$$W(t) = \int_0^t M(t) \left\{ \frac{1}{I} \int_0^t [M(t) - M(\theta)]\, dt \right\} dt \qquad [14]$$

In order to calculate the total work done this last expression has to be integrated up to the time at which the maximum deflection takes place. If the time at which this maximum deflection takes place is much greater than the time of duration of the load, T, then we only have to integrate up to T. Under such circumstances, $M(\theta)$ will be small compared to $M(t)$ during the time 0 to T. Practical structures such as

fuselages and wings under blast at distances of 10' - 50'* will fall into this catagory. Thus

$$W = \int_0^T M(t)\left[\frac{1}{I}\int_0^t M(t)\,dt\right]dt \qquad [15]$$

or

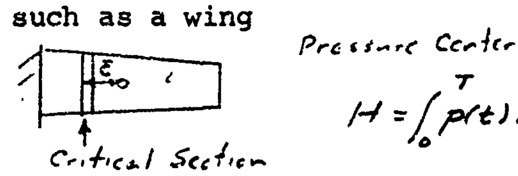
$$W = \frac{H^2}{2I} \qquad [16]$$

where

$$H = \int_0^T M(t)\,dt \qquad [17]$$

The energy available from the blast field is $H^2/2I$ (i.e. the external work) and this is equated to the energy absorbed by the structure, $\int_0^{\theta} M(\theta)\,d\theta$ which is the internal work.

For a flat section such as a wing



$$H = \int_0^T p(t) A \bar{c}\,dt = A\bar{c}\bar{I} \qquad [18]$$

where $p(t)$ = blast pressure (assumed uniform)

A = surface area of wing from critical section to edge

$\bar{c}$ = distance from critical section to pressure center

$\bar{I} = \int_0^T p(t)\,dt$ = applied impulse

and

$$I = \frac{M\ell^2}{3}, \quad M = \mu A \qquad \mu = \text{mass/unit area} \qquad [19]$$

Thus for the wing or other flat control surface

$$\frac{H^2}{2I} = 3\left(\frac{\bar{c}}{\ell}\right)^2\left(\frac{\bar{I}^2}{2\mu}A\right) \qquad [20]$$

For the fuselage, the loading tending to produce moment is acting only over half the surface area (i.e. over the half facing the blast). Thus

$$H = \int_0^T p(t)\frac{A}{2}\bar{c}\,dt = \frac{A}{2}\bar{c}\bar{I} \qquad [21]$$

$$\frac{H^2}{2I} = \frac{3}{4}\left(\frac{\bar{c}}{\ell}\right)^2\left(\frac{\bar{I}^2}{2\mu}A\right) \qquad [22]$$

In cases where the wing or fuselage is loaded with a static loading such as fuel, end tanks, etc. there is an additional turning moment present. Once the structure is weakened by the fragments taking out material at a given section, the blast arrives and can do additional damage (i.e. produce a larger hinge angle around the critical section) if it contains sufficient energy. After this deformation is completed, the static moment can then produce further hinging if it is of sufficient magnitude. This type of loading will be discussed in the sample problem given later in this report.

*i.e. for frag-blast type weapons

V. Calculation of the internal work

A. Flat surfaces

The internal work consists of the elastic work done on the structure bringing it up to failure plus the plastic work done by the hinge restraining moment at the critical section. In wings, elevators and other control surfaces which are cantilevered from the fuselage, the failure takes place by way of a hinging at a weakened section.[5] This weakened section is formed when a portion of the structure such as skin or spar caps, etc. is knocked out by fragments. If the fragments perforate across the entire surface, thus cutting the surface into two parts, then this will constitute kill by itself without any further calculations. If only a portion of the section is knocked out we can then calculate how much additional damage will be done by blast.

Given a typical wing section such as shown in Figure 19

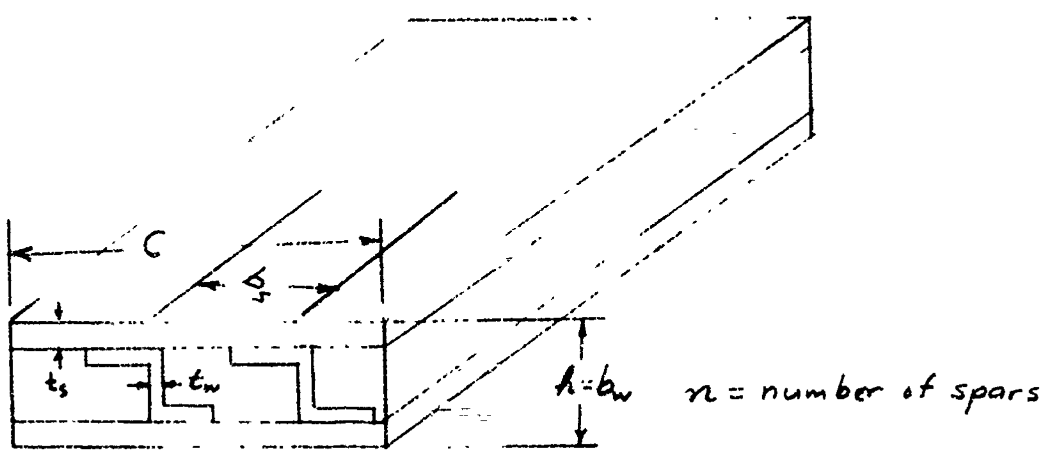


Fig. 19 Typical Wing Section

First the failure moment is computed in accordance with the relation

$$M_{fail} = I\,\sigma_f / h/2 \qquad [23]$$

where I is the area moment of inertia of the weakened section (after perforation takes place) and h/2 is the half depth of the control surface.

$$\sigma_f = \text{Failure Stress} = \eta\,\frac{k_f \pi^2 E}{12(1-\nu^2)}\left(\frac{t_s}{b_s}\right)^2 \qquad [24]$$

where η is the plasticity reduction factor obtained from a previous reference,[5] E is the modulus of elasticity, ν is Poisson's ratio, t_s is the thickness of the cover skin (see Fig.19), b_s is the width of the cover skin between rivet lines attaching skin to flanges and

$$k_f = \sqrt{\frac{\frac{48}{\pi^4}\left(3\frac{f}{b_w}+1\right)}{\left(\frac{f}{b_w}\;\frac{b_w/t_w}{b_s/t_s}\right)^3\left(3\frac{f}{b_w}+4\right)}} \qquad [25]$$

where $b_w = h$

t_w = web thickness (see Fig.19)

f = distance from centerline of web to the point where the rivet clamps the flange to the cover skin

After failure has occurred, the moment - hinge angle relation is given by the following equation:

$$M = m_{ow} h n (2\psi_1 + \psi_2) + C m_{os}\psi_3 + m_{ow}\frac{h^2}{r} n\,\psi_4 \qquad [26]$$

where m_{ow} = plastic moment per unit length of the web = $\sigma_{ow} t_w^2/4$

in which σ_{ow} = yield stress of web

t_w = thickness of web

h = depth of beam

n = number of webs in cross section

c = width of cover skin

m_{os} = plastic moment per unit width of cover skin = $\sigma_{os} t_s^2/4$

where σ_{os} = yield stress of cover skin

t_s = thickness of cover skin

r = web folding radius (see D'Amato[5])

The functions $\psi_1, \psi_2, \psi_3, \psi_4$ are dependent upon the hinge angle θ and are as follows:

$$\psi_1(\theta) = \frac{(\phi^2+1)^{3/2}}{\phi(1+\cos\theta+\phi\sin\theta)\sqrt{\frac{2(\phi+1/\phi)\sin\theta+(\phi^2-1/\phi^2)(1-\cos\theta)}{1+\cos\theta}}}$$

$$\psi_2(\theta) = \frac{(\phi^2-1)\sin\theta+2\phi\cos\theta}{\sqrt{\phi^4-\left[1-(\phi^2-1)\cos\theta-2\phi\sin\theta\right]^2}}$$

$$\psi_3(\theta) = \frac{2(\sin\theta/2 + 1/\phi \cos\theta/2)}{\sqrt{1-(\cos\theta/2 - 1/\phi \sin\theta/2)^2}}$$

$$\psi_4(\theta) = \phi \frac{\cos\Lambda}{\sin\Lambda} \frac{1-\sin\Lambda}{1+\sin\Lambda}(\phi\cos\theta/2 - \sin\theta/2)$$

$$\phi = \lambda'/2k \qquad [27]$$

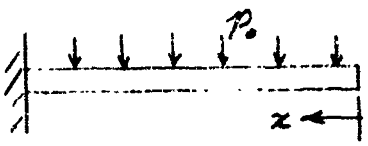
$$\lambda' = b_s\sqrt{2/t_1}$$

$$\Lambda = \cos^{-1}\frac{1}{\phi\sin\theta/2 + \cos\theta/2}$$

The elastic energy absorbed by the structure can be written[3]

$$U = \int_0^{l} \frac{M^2 dx}{2EI} \qquad [28]$$

where M is the bending moment in the stsucture, E is the modulus of elasticity and I is the area moment of inertia.



For a uniformly loaded cantilever beam such as shown above, the bending moment is as follows:

$$M = \tfrac{1}{2}\, p_0 x^2$$

The maximum moment will occur at the root and will be

$$M_0 = \tfrac{1}{2}\, p_0 l^2$$

Thus

$$M = M_0 (x/l)^2 \qquad [29]$$

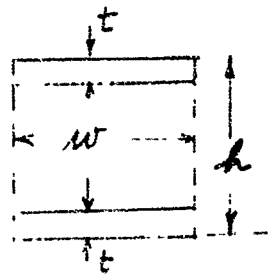
and the energy absorbed elastically will be

$$U = M_0^2 l / 10EI \qquad [30]$$

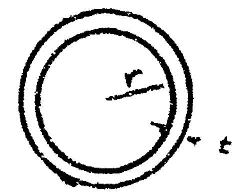
The plastic energy absorbed will be the area under the M - θ curve in the plastic region shown in Figure 18

B. Cylindrical shells

We can use D'Amato's theory[5] for the fuselage by making an equivalent flat structure from the fuselage. To do this we equate the load carrying area in the top and bottom plates of a flat structure to the load carrying area of the cylindrical shell of the same thickness.



Equivalent Flat Structure



Cylinder

Thus

$$2tw = 2\pi r t \qquad [31]$$
$$w = \pi r$$

The width of the flat structure will be ½ of the circumference of the cylinder.

The height, h, of the equivalent flat structure is found by equating the area moments of inertia of the flat structure and cylinder thus

$$2tw(h/2)^2 = \pi r^3 t$$

and

$$h = \sqrt{2}\, r \qquad [32]$$

The equivalent thickness of the fuselage which contains plating and attached stringers will be taken as

$$t_e = \frac{An}{\pi D} + t_o \qquad [33]$$

where A = cross sectional area of stiffeners
n = number of longitudinals in cross section
t_e = euqivalent uniform thickness of cylinder
t_o = thickness of skin itself

VI. Sample problem

A typical wing and fuselage section were considered. The blast energy available for 6.5# of explosive as a function of distance is given in Fig. 20 for the wing section. Impulse values given by Goodman[4] were used. The energy absorption (or internal work) curves for the structure are shown in Figure 21. In this particular case there was a static loading on the wing having a moment of 1,000,000 # in. around the critical section (Sta. 143). Figs. 20 and 21 show that the energy content of the 6.5# blast is so small that it would do practically no damage to the wing even if 80% of the wing was knocked out by fragments. Figures 22 and 23 show that the static moment is so large that it will fail the wing after the fragments have done their damage. The failure moment is shown in Fig. 22. This figure shows that the static moment will not inflict damage until about 70" of structure is knocked out at the critical section. Fig. 23 illustrates that the failure will be complete since the resisting moment for a structure with 70" removed is less than half of the static moment.

Fig. 24 illustrates the energy available in the 6.5# blast on the fuselage and Fig. 25 shows the internal work curves for the fuselage section near the tail. These curves show that if most of the circumference is knocked out by the fragments then the 6.5# blast very close to the fuselage (less than 5 feet) might produce a small post failure hinge angle.

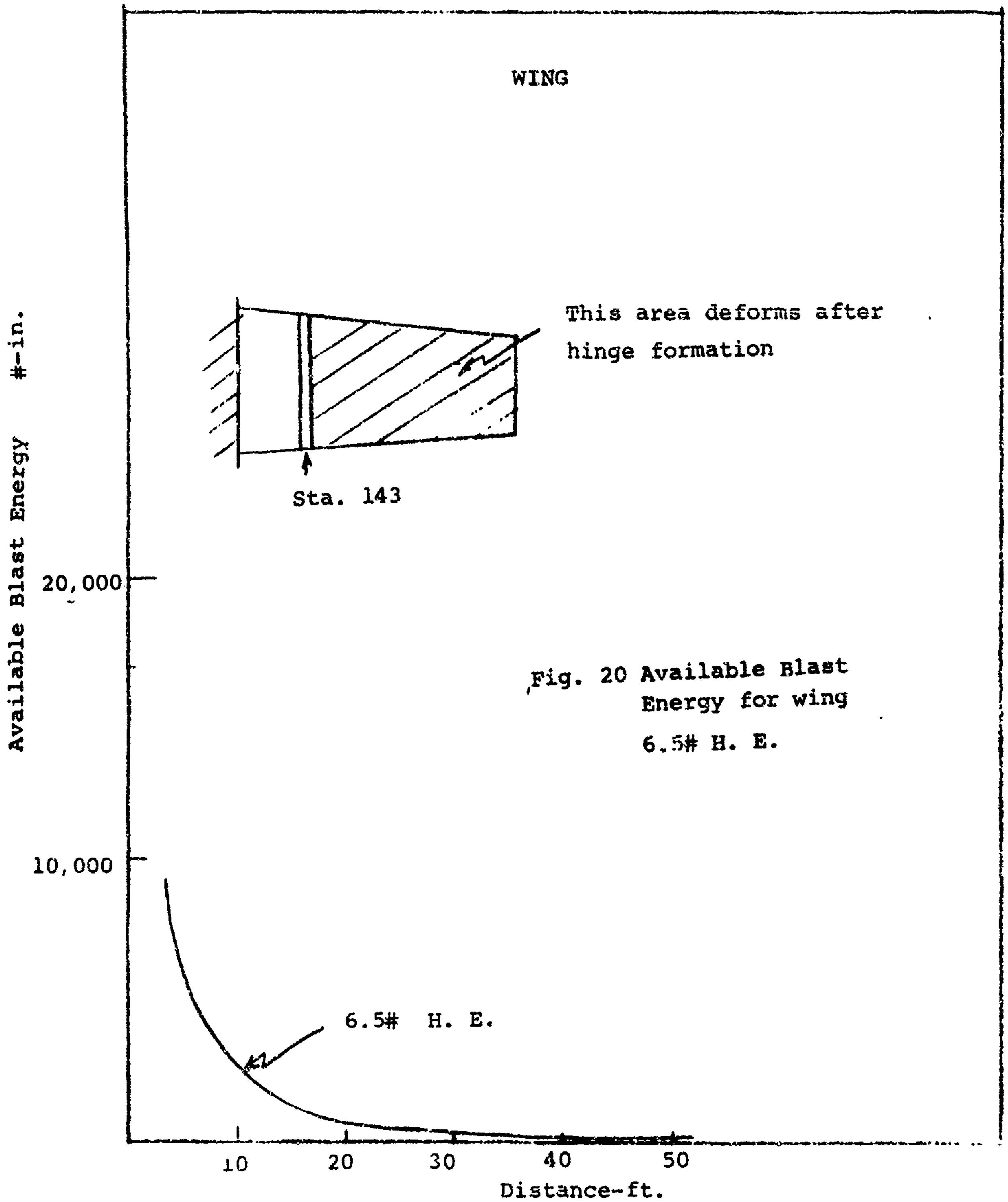


Fig. 20 Available Blast Energy for wing 6.5# H. E.

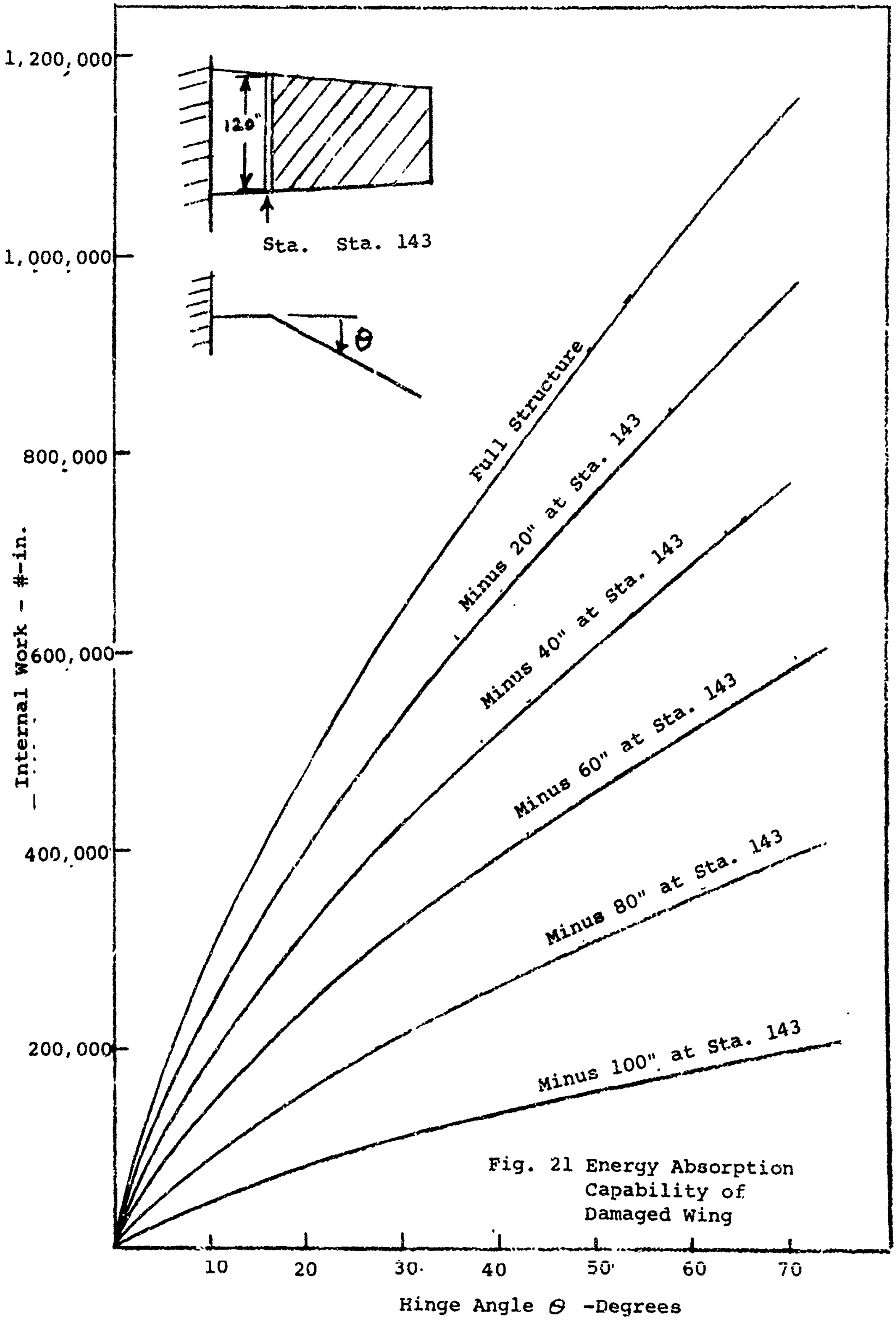


Fig. 21 Energy Absorption Capability of Damaged Wing

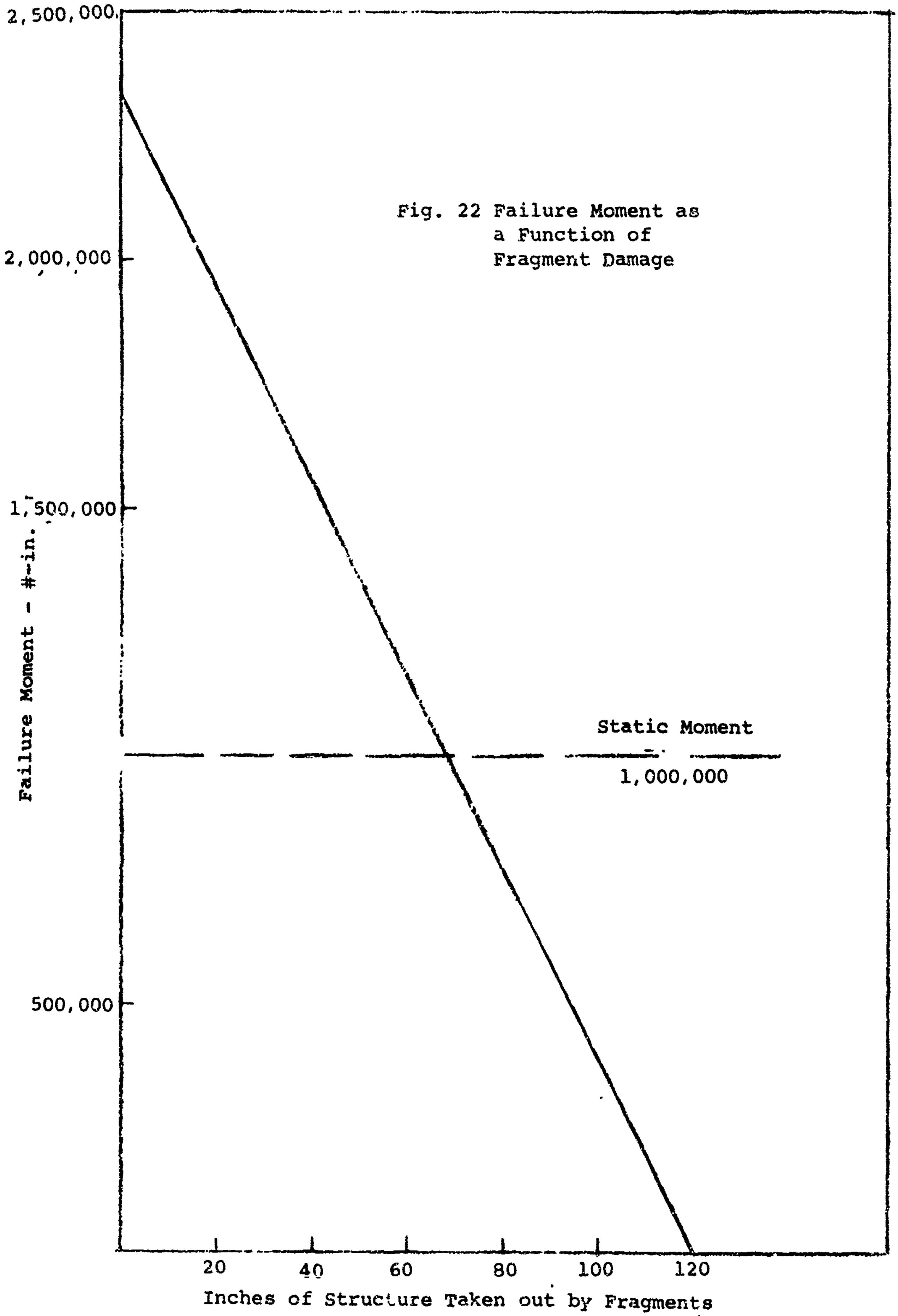
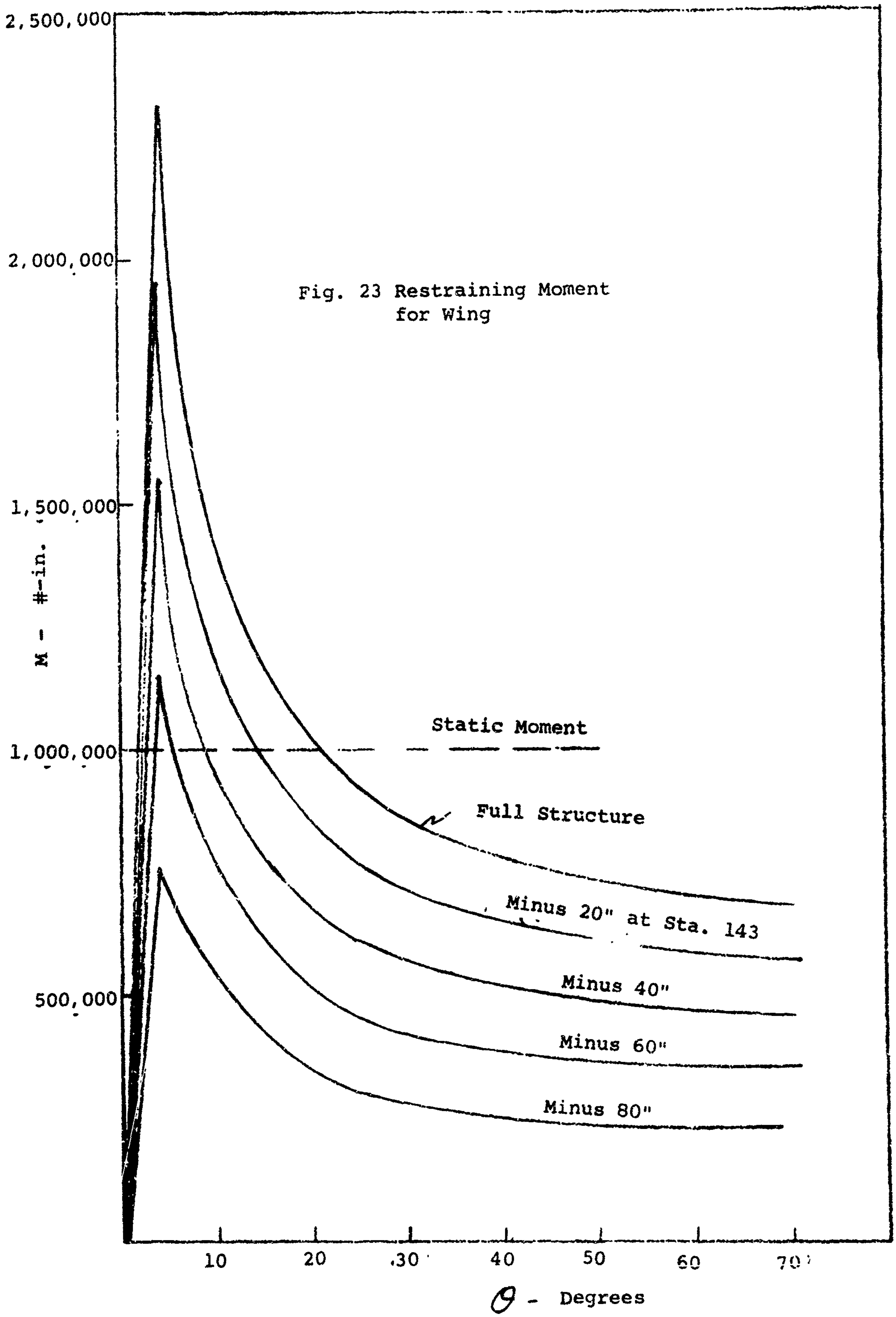


Fig. 22 Failure Moment as a Function of Fragment Damage


2,500,000
2,000,000
1,500,000
1,000,000
500,000
M - #-in.
Fig. 23 Restraining Moment
for Wing
Static Moment
Full Structure
Minus 20" at Sta. 143
Minus 40"
Minus 60"
Minus 80"
10
20
30
40
50
60
70
θ - Degrees

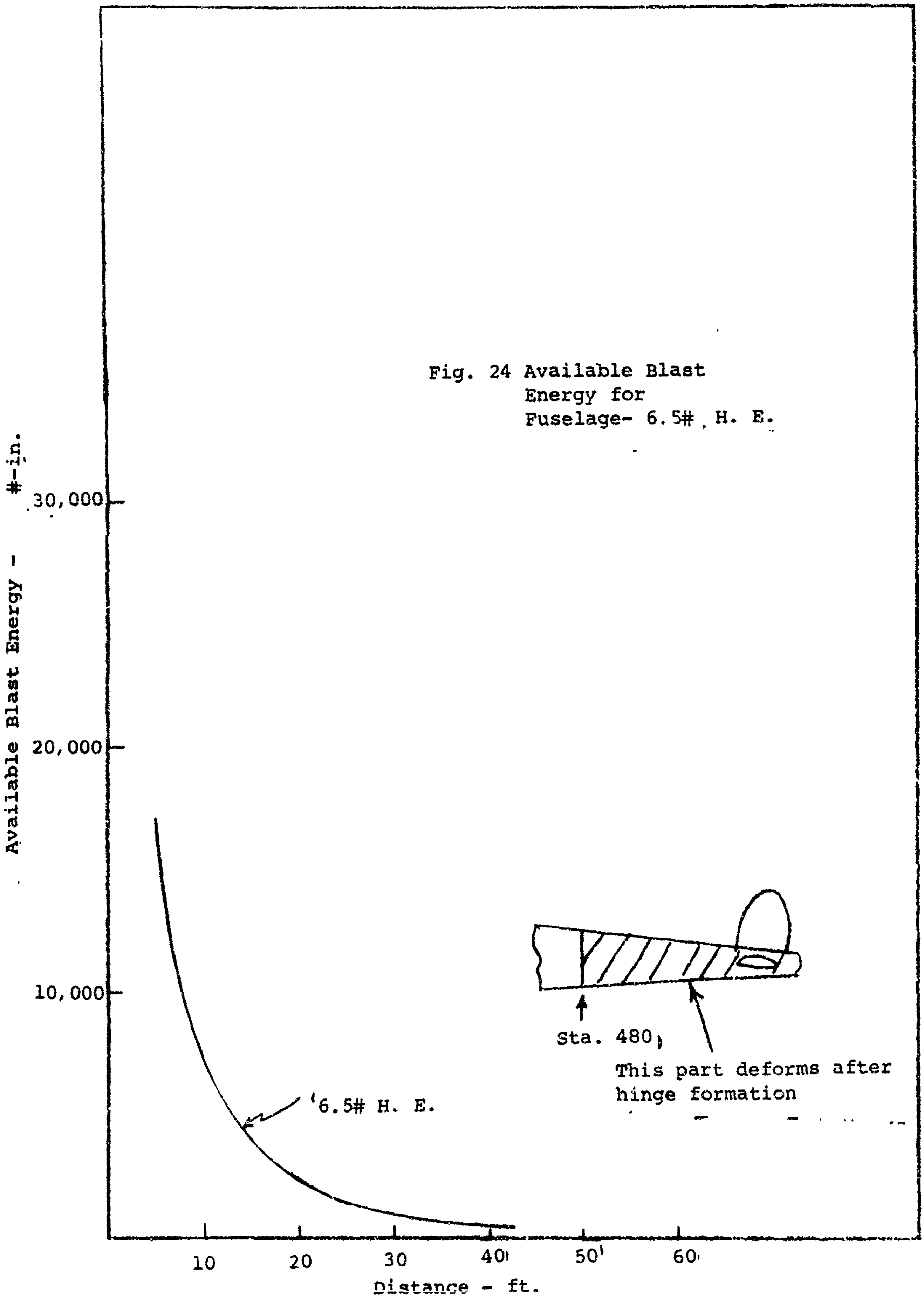


Fig. 24 Available Blast Energy for Fuselage- 6.5# H. E.

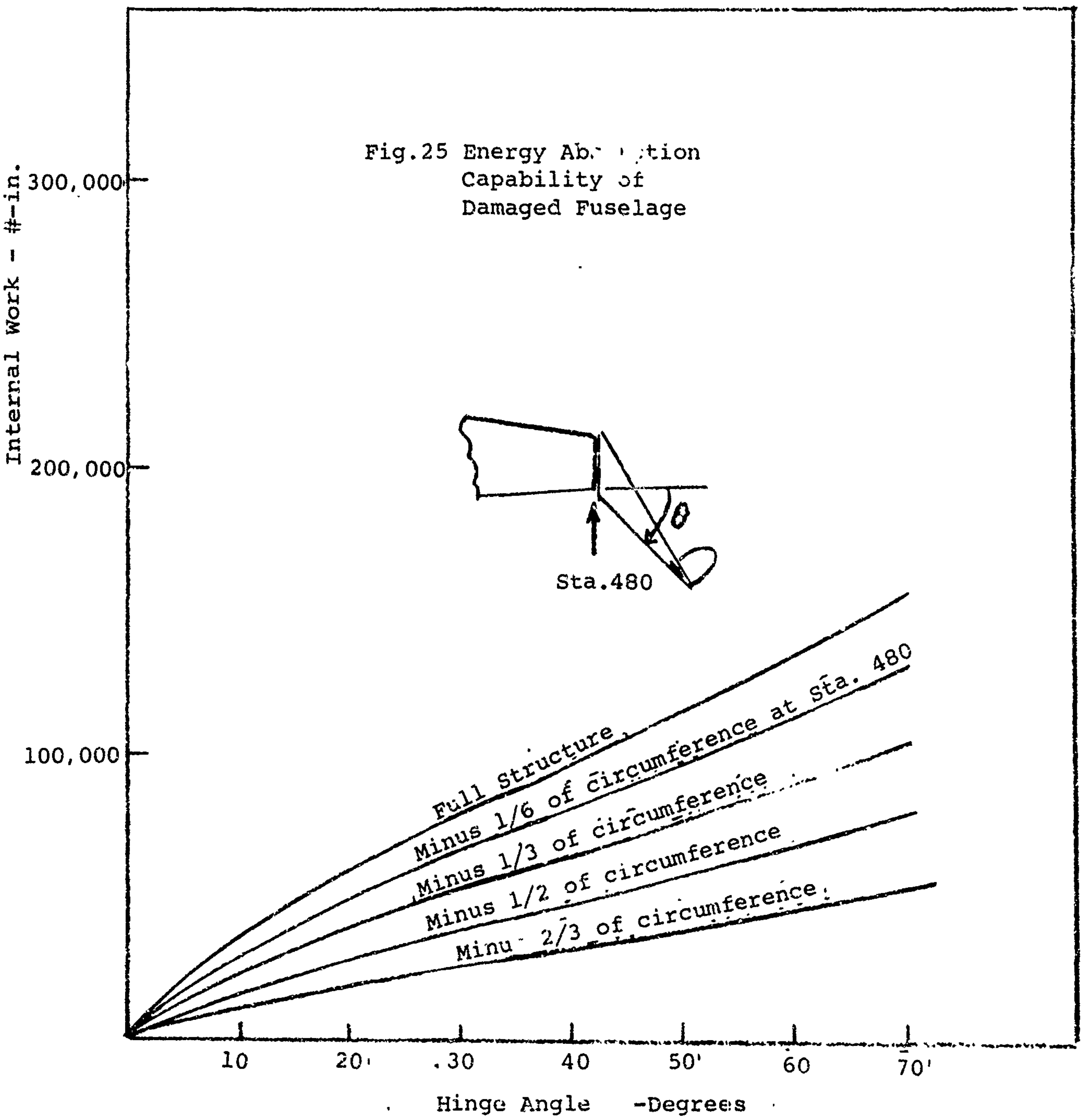


Fig.25 Energy Absorption Capability of Damaged Fuselage

VII. Determination of modes of failure for monocoque structures under pure blast

Experiments have shown[10-12] that monocoque cylindrical shells subject to side on blast will fail in one of two types of modes. The first is the collapse mode as seen in Figure 9 and the second is lobar buckling as seen in Figure 9a. There is a controversy as to whether the collapse is an instability known as collapse hinge buckling[13-14], or whether it is a straight collapse determined by the criterion that the Von Mises yield condition is satisfied at the maximum stress points. Figure 26 shows curves computed for the three types of failure. The curves for collapse hinge buckling and lobar buckling were taken from Reference 9. In these curves L is the shell length, a is the radius and h is the thickness. The collapse curves were computed using membrane theory[2] as follows:

If N_φ is the peripheral membrane stress resultant, N_x the longitudinal membrane stress resultant and $N_{x\varphi}$ the membrane shear stress resultant then

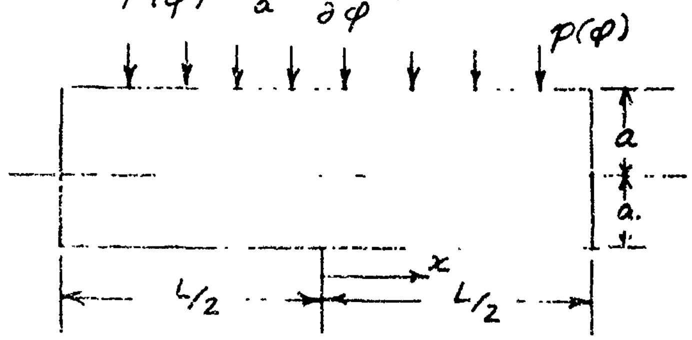
$$N_\varphi = a\,p(\varphi)$$

where $p(\varphi)$ is the peripheral pressure distribution

$$N_x = -\frac{L}{8a}(\ell^2 - 4x^2)\frac{dF(\varphi)}{d\varphi} \qquad [34]$$

$$N_{x\varphi} = -x\,F(\varphi)$$

$$F(\varphi) = \frac{1}{a}\frac{\partial N_\varphi}{\partial \varphi}$$



Let the length of the shell be L and assume that it is supported by a diaphragm at each end $x = \pm L/2$. If the origin is at the center of the shell the boundary conditions are

$$N_x = 0 \quad @ \quad x = \pm L/2 \qquad [35]$$

The pressure used in computing the curves of Ref. 9 (Fig. 26) was

$$\begin{aligned} p(\varphi) &= (P_R - P_I)\cos^2\varphi + P_I \quad &\text{for } \tfrac{3\pi}{2} \le \varphi \le \tfrac{\pi}{2} \\ &= P_I &\tfrac{\pi}{2} \le \varphi \le \tfrac{3\pi}{2} \end{aligned} \qquad [36]$$

Thus

$$N_\varphi = a(P_R - P_I)\cos^2\varphi + a\,P_I$$
$$F(\varphi) = -(P_R - P_I)\sin 2\varphi \qquad [37]$$
$$N_{x\varphi} = x\,(P_R - P_I)\sin 2\varphi$$
$$N_x = \frac{1}{8a}(\ell^2 - 4x^2)(P_R - P_I)\,2\cos 2\varphi$$

Plasticity will first begin at $x = \varphi = 0$ if the Von Mises yield condition is satisfied at this point, i.e.

$$N_x^2 - N_x N_\varphi + N_\varphi^2 + 3N_{x\varphi}^2 = \sigma_s^2 h^2 \qquad [38]$$

where σ_s = yield stress in pure tension

Thus the equation for the determination of the critical value of P_R (or P_I) is

$$\frac{L^4}{16a^4}(P_R - P_I)^2 - \frac{L^2}{4a^2}P_R(P_R - P_I) + P_R^2 = \frac{\sigma_s^2 h^2}{a^2} \qquad [39]$$

Given the ratio P_R/P_I, the critical value of P_I can be determined. The value of q plotted in Fig. 26 is

$$q = \frac{P_I\,a(1-\nu^2)}{E h} \qquad [40]$$

The values computed from the above equation for $P_R/P_I = 2, 5$ are shown in Figure 26 along with the values for collapse hinge buckling and lobar buckling given in Reference 9. The collapse curves were computed using $\nu = .3$ and $E/\sigma_s = 1000$, which seemed realistic for steel cylinders.

Among Schuman's tests[10,11] there were seven steel cylinders, each with $a/h = 79$ and L/a varying from 4 to 16. All seven cylinders went into the collapse mode of failure. Examination of Figure 26 reveals that for L/a between 4 and 16 collapse would occur before lobar buckling and lobar buckling would occur before collapse hinge buckling. Since the tests showed collapse patterns, it can be concluded that all the cylinders failed in ordinary collapse as described by the theory in this section of the report. Extrapolation of the curves for $a/h = 79$ from the value of $a/h = 64.6$ shown in Figure 26 will change the curves only slightly.

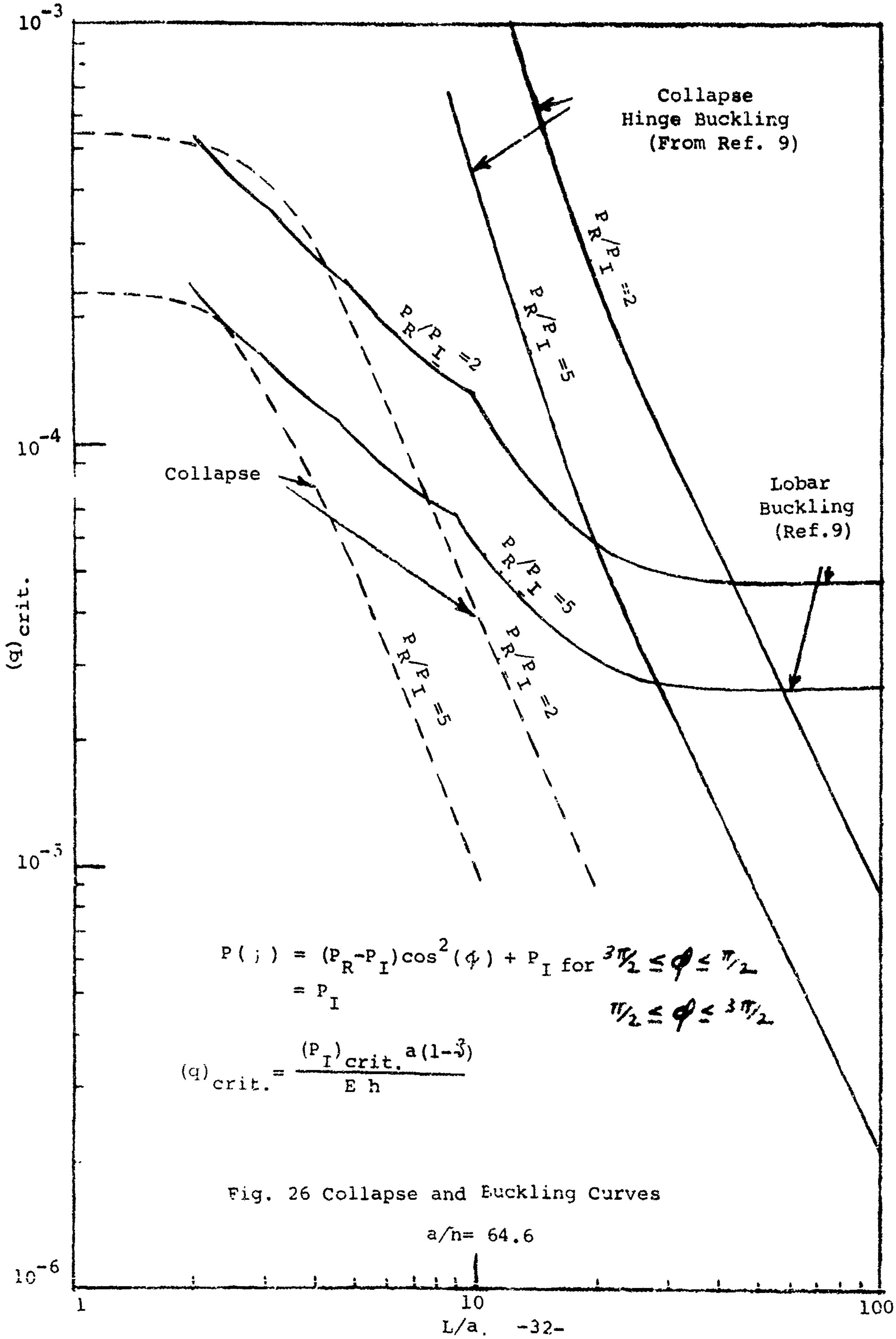


Fig. 26 Collapse and Buckling Curves

a/h= 64.6

VIII. Critical non-dimensional parameters in fragment-blast response

A. Perforation

In section III A the basic empirical equations for perforation were given. A theoretical relation for residual velocity is given in Goldsmith's book[15] and is as follows:

$$V_r = V_s \, e^{-\rho A h_o / m} \qquad [41]$$

where V_r = residual velocity after penetration

V_s = initial velocity

ρ = mass density of plate being perforated

A = frontal area being perforated

h_o = thickness of plate being perforated

m = mass of the projectile doing the perforating

The quantity $\rho A h_o$ is the mass of the plug which is taken out of the plate. Therefore the above equation becomes

$$\frac{V_r}{V_s} = e^{-m_p/m} \qquad [42]$$

Thus the ratio of residual to initial velocity is dependent upon the ratio of the mass of the plug taken out of the plate to the mass of the projectile perforating the plate.

Figure 27 shows a comparison between the results obtained from equations [3] and [41]. It is seen that the results compare reasonably well. Thus the important dimensionless ratio in determining residual velocity is m_p/m.

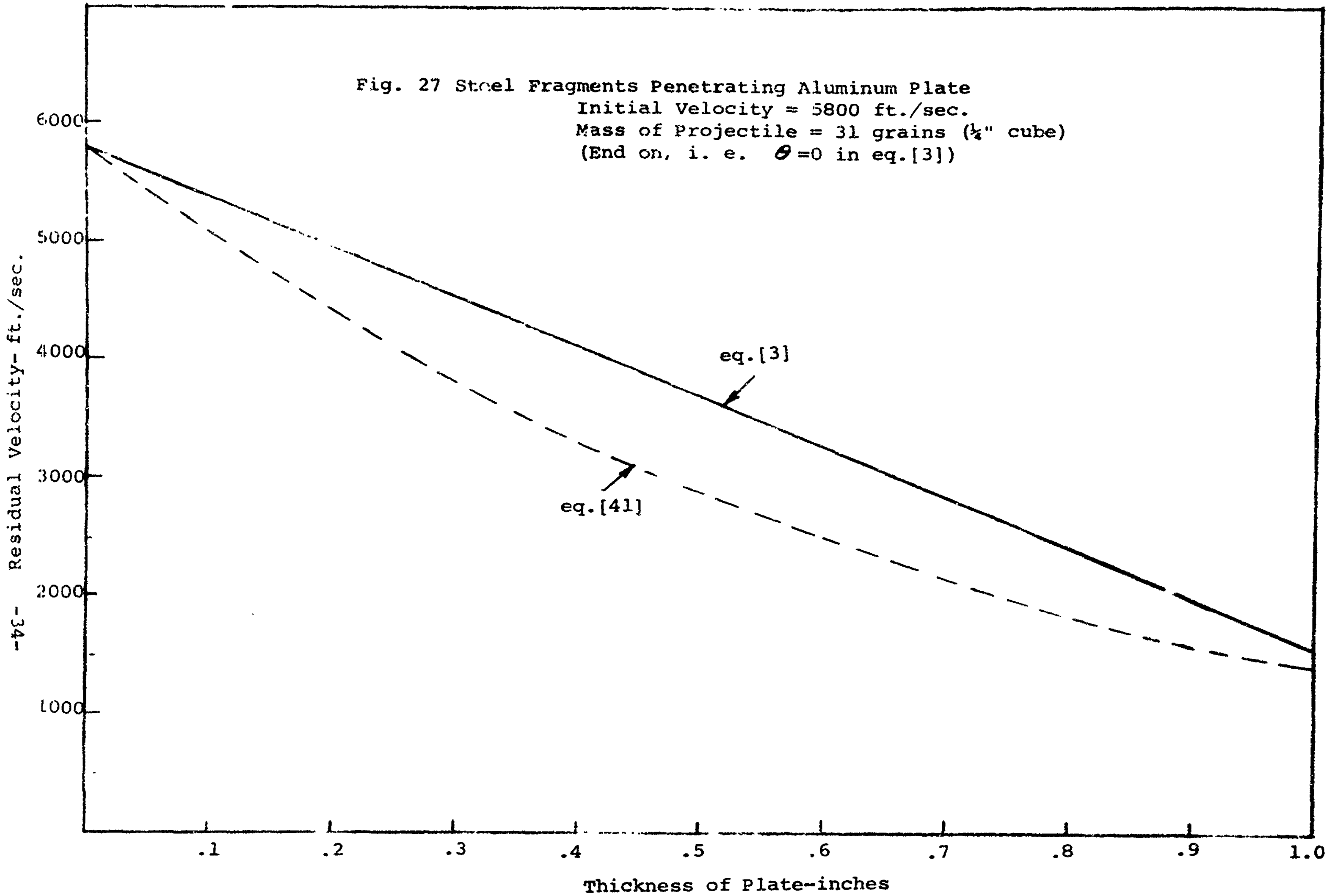


Fig. 27 Steel Fragments Penetrating Aluminum Plate
Initial Velocity = 5800 ft./sec.
Mass of Projectile = 31 grains (¼" cube)
(End on, i. e. $\theta = 0$ in eq.[3])

B. Post failure buckling

1. Fuselages

In section VB of this report we assumed that once the weakened section was formed by perforation, the fuselage would then buckle at the weakened section. The moment-hinge angle (M-θ) relation for the fuselage was determined by the equation

$$M = C m_{os} \psi_3 \qquad \text{(from eq. [26])}$$

where
$$C = \omega r = \pi r \quad \text{(from eq. [31])}$$
$$m_{os} = \sigma_o t_s^2/4 \qquad [43]$$
$$t_s = t_e = \frac{An}{\pi D} + t_o$$

From equation [27] it is seen that the value of ψ_3 depends upon the hinge angle θ and $\varphi(\lambda'/2h)$, where λ' is the wave length of the buckle and h is the depth of the beam. The value of ψ_3 for various values of F ($=\varphi$) is given in Table 2 along with the values of ψ_3 integrated over θ (i.e. $\int_0^{\theta_0} \psi_3(F,\theta)d\theta$). The energy absorbed at the hinge is

$$E = C m_{os} \int_0^{\theta_0} \psi_3(F,\theta)\,d\theta \qquad [44]$$

Using Table 2 and the relations given previously, the M-θ curve and the energy absorption can be readily computed.

2. Wings

For the wing section the post failure moment depends upon ψ_1, ψ_2, ψ_3, ψ_4 and the energy absorption at the hinge will be

$$E = m_{ow} h n \int_0^{\theta_0} (2\psi_1 + \psi_2)\,d\theta + C m_{os} \int_0^{\theta_0} \psi_3\,d\theta + m_{ow} \frac{h^2}{r} n \int_0^{\theta_0} \psi_4\,d\theta \qquad [45]$$

Table 2 contains $\psi_1, \psi_2, \psi_3, \psi_4$ as functions of F($=\varphi$) and θ as well as $\int\psi_1 d\theta, \int\psi_2 d\theta, \int\psi_3 d\theta, \int\psi_4 d\theta$. The nomenclature in Table 2 is as follows:

$$\psi_1 = P1 \qquad \int\psi_1 d\theta = V6 \qquad [46]$$
$$\psi_2 = P2 \qquad \int\psi_2 d\theta = V7 \qquad \theta_o = J \text{ (hinge angle)}$$
$$\psi_3 = P3 \qquad \int\psi_3 d\theta = V8$$
$$\psi_4 = P4 \qquad \int\psi_4 d\theta = V9$$

Table 2. Nondimensional Buckling Functions

F= 0.5
J= 5 P1= 2.36659 P2= 4.97939 P3= 9.95878 P4= 0.740835
V6= 0.205893 V7= 0.433207 V8= 0.866414 V9= 6.44004E-2
F= 0.5
J= 10 P1= 1.85062 P2= 3.65528 P3= 7.31056 P4= 0.41838
V6= 0.366897 V7= 0.751216 V8= 1.50243 V9= 0.100272
F= 0.5
J= 15 P1= 1.57828 P2= 3.09786 P3= 6.19572 P4= 0.2744
V6= 0.504207 V7= 1.02073 V8= 2.04146 V9= 0.124145
F= 0.5
J= 20 P1= 1.40842 P2= 2.78485 P3= 5.56969 P4= 0.195696
V6= 0.626739 V7= 1.26301 V8= 2.52602 V9= 0.141171
F= 0.5
J= 25 P1= 1.29386 P2= 2.58629 P3= 5.17257 P4= 0.143706
V6= 0.739305 V7= 1.48802 V8= 2.97604 V9= 0.153673
F= 0.5
J= 30 P1= 1.21387 P2= 2.45273 P3= 4.90546 P4= 0.10602
V6= 0.844913 V7= 1.70141 V8= 3.40281 V9= 0.162897
F= 0.5
J= 35 P1= 1.15784 P2= 2.36102 P3= 4.72205 P4= 7.67278E-2
V6= 0.945644 V7= 1.90682 V8= 3.81363 V9= 0.169572
F= 0.5
J= 40 P1= 1.11987 P2= 2.29898 P3= 4.59795 P4= 5.25781E-2
V6= 1.04307 V7= 2.10683 V8= 4.21365 V9= 0.174147
F= 0.5
J= 45 P1= 1.0966 P2= 2.2598 P3= 4.5196 P4= 3.15593E-2
V6= 1.13848 V7= 2.30343 V8= 4.60686 V9= 0.176892
F= 0.5
J= 50 P1= 1.08618 P2= 2.23975 P3= 4.4795 P4= 0.01228
V6= 1.23297 V7= 2.49829 V8= 4.99657 V9= 0.177961

F= 1
J= 5 P1= 1.65491 P2= 3.53677 P3= 7.07355 P4= 1.81264
V6= 0.143977 V7= 0.307699 V8= 0.615399 V9= 0.157699
F= 1
J= 10 P1= 1.42574 P2= 2.60312 P3= 5.20624 P4= 0.972366
V6= 0.268016 V7= 0.534171 V8= 1.06834 V9= 0.242295
F= 1
J= 15 P1= 1.26139 P2= 2.208 P3= 4.416 P4= 0.638516
V6= 0.377757 V7= 0.726267 V8= 1.45253 V9= 0.297846
F= 1
J= 20 P1= 1.13778 P2= 1.98303 P3= 3.96605 P4= 0.458095
V6= 0.476744 V7= 0.89879 V8= 1.79758 V9= 0.3377
F= 1
J= 25 P1= 1.04186 P2= 1.83657 P3= 3.67314 P4= 0.345574
V6= 0.567386 V7= 1.05857 V8= 2.11714 V9= 0.367765
F= 1
J= 30 P1= 0.965839 P2= 1.73365 P3= 3.46731 P4= 0.269123
V6= 0.651414 V7= 1.2094 V8= 2.4188 V9= 0.391179
F= 1
J= 35 P1= 0.904749 P2= 1.65774 P3= 3.31548 P4= 0.214054
V6= 0.730127 V7= 1.35362 V8= 2.70725 V9= 0.409808
F= 1
J= 40 P1= 0.855266 P2= 1.5999 P3= 3.19981 P4= 0.172623
V6= 0.804535 V7= 1.49281 V8= 2.98563 V9= 0.42482
F= 1
J= 45 P1= 0.815082 P2= 1.55487 P3= 3.10974 P4= 0.14035
V6= 0.875448 V7= 1.62809 V8= 3.25618 V9= 0.43703
F= 1
J= 50 P1= 0.782553 P2= 1.51932 P3= 3.03864 P4= 0.114463
V6= 0.94353 V7= 1.76027 V8= 3.52054 V9= 0.446988

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F= 1.5
J= 5 P1= 1.60181 P2= 2.92162 P3= 5.84323 P4= 2.93596
V6= 0.139357 V7= 0.254181 V8= 0.508361 V9= 0.255428
F= 1.5
J= 10 P1= 1.41703 P2= 2.17159 P3= 4.34317 P4= 1.50329
V6= 0.262638 V7= 0.443108 V8= 0.886217 V9= 0.386214
F= 1.5
J= 15 P1= 1.27103 P2= 1.8573 P3= 3.7146 P4= 0.955934
V6= 0.373218 V7= 0.604693 V8= 1.20939 V9= 0.469381
F= 1.5
J= 20 P1= 1.15353 P2= 1.67974 P3= 3.35948 P4= 0.669747
V6= 0.473575 V7= 0.750831 V8= 1.50166 V9= 0.527649
F= 1.5
J= 25 P1= 1.05761 P2= 1.5648 P3= 3.1296 P4= 0.496506
V6= 0.565587 V7= 0.886968 V8= 1.77394 V9= 0.570845
F= 1.5
J= 30 P1= 0.978448 P2= 1.4843 P3= 2.9686 P4= 0.382055
V6= 0.650712 V7= 1.0161 V8= 2.0322 V9= 0.604083
F= 1.5
J= 35 P1= 0.912556 P2= 1.42494 P3= 2.84989 P4= 0.301836
V6= 0.730104 V7= 1.14007 V8= 2.28014 V9= 0.630343
F= 1.5
J= 40 P1= 0.857384 P2= 1.37959 P3= 2.75918 P4= 0.243116
V6= 0.804697 V7= 1.2601 V8= 2.52019 V9= 0.651494
F= 1.5
J= 45 P1= 0.811017 P2= 1.34402 P3= 2.68803 P4= 0.198662
V6= 0.875255 V7= 1.37703 V8= 2.75405 V9= 0.668778
F= 1.5
J= 50 P1= 0.771999 P2= 1.31558 P3= 2.63115 P4= 0.164072
V6= 0.942419 V7= 1.49148 V8= 2.98296 V9= 0.683052

F= 3
J= 5 P1= 2.13214 P2= 2.15077 P3= 4.30155 P4= 6.13254
V6= 0.185496 V7= 0.187117 V8= 0.374235 V9= 0.536141
F= 3
J= 10 P1= 1.86115 P2= 1.6507 P3= 3.30141 P4= 2.80658
V6= 0.347416 V7= 0.330729 V8= 0.661457 V9= 0.780314
F= 3
J= 15 P1= 1.64765 P2= 1.44892 P3= 2.89784 P4= 1.64011
V6= 0.490761 V7= 0.456785 V8= 0.913569 V9= 0.923004
F= 3
J= 20 P1= 1.47617 P2= 1.33861 P3= 2.67722 P4= 1.07518
V6= 0.619188 V7= 0.573244 V8= 1.14649 V9= 1.01654
F= 3
J= 25 P1= 1.3363 P2= 1.26924 P3= 2.53848 P4= 0.754956
V6= 0.735447 V7= 0.683667 V8= 1.36733 V9= 1.08223
F= 3
J= 30 P1= 1.22076 P2= 1.22187 P3= 2.44374 P4= 0.555351
V6= 0.841653 V7= 0.78997 V8= 1.57994 V9= 1.13054
F= 3
J= 35 P1= 1.12431 P2= 1.1877 P3= 2.3754 P4= 0.422606
V6= 0.939468 V7= 0.8933 V8= 1.7866 V9= 1.16731
F= 3
J= 40 P1= 1.04312 P2= 1.16207 P3= 2.32414 P4= 0.330025
V6= 1.03022 V7= 0.9944 V8= 1.9888 V9= 1.19602
F= 3
J= 45 P1= 0.974316 P2= 1.14227 P3= 2.28454 P4= 0.263037
V6= 1.11499 V7= 1.09378 V8= 2.18756 V9= 1.2189
F= 3
J= 50 P1= 0.915697 P2= 1.12662 P3= 2.25324 P4= 0.213114
V6= 1.19465 V7= 1.19179 V8= 2.38359 V9= 1.23744
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F= 5
J= 5 P1= 3.02823 P2= 1.75614 P3= 3.51229 P4= 9.8059
V6= 0.263456 V7= 0.152785 V8= 0.305569 V9= 0.853113
F= 5
J= 10 P1= 2.53145 P2= 1.39831 P3= 2.79663 P4= 3.97224
V6= 0.483692 V7= 0.274438 V8= 0.548875 V9= 1.1987
F= 5
J= 15 P1= 2.17001 P2= 1.26071 P3= 2.52143 P4= 2.13699
V6= 0.672483 V7= 0.38412 V8= 0.76824 V9= 1.38462
F= 5
J= 20 P1= 1.89637 P2= 1.18847 P3= 2.37695 P4= 1.314
V6= 0.837467 V7= 0.487517 V8= 0.975034 V9= 1.49893
F= 5
J= 25 P1= 1.68292 P2= 1.1446 P3= 2.2892 P4= 0.876079
V6= 0.983881 V7= 0.587098 V8= 1.17419 V9= 1.57515
F= 5
J= 30 P1= 1.51253 P2= 1.11554 P3= 2.23108 P4= 0.617331
V6= 1.11547 V7= 0.684149 V8= 1.3683 V9= 1.62886
F= 5
J= 35 P1= 1.37401 P2= 1.09513 P3= 2.19025 P4= 0.453034
V6= 1.23501 V7= 0.779425 V8= 1.55885 V9= 1.66827
F= 5
J= 40 P1= 1.25973 P2= 1.08016 P3= 2.16033 P4= 0.34301
V6= 1.34461 V7= 0.8734 V8= 1.7466 V9= 1.69812
F= 5
J= 45 P1= 1.16435 P2= 1.06884 P3= 2.13768 P4= 0.266228
V6= 1.4459 V7= 0.966389 V8= 1.93278 V9= 1.72128
F= 5
J= 50 P1= 1.08397 P2= 1.06005 P3= 2.1201 P4= 0.210838
V6= 1.54021 V7= 1.05861 V8= 2.11723 V9= 1.73962

F= 10
J= 5 P1= 4.96209 P2= 1.3953 P3= 2.7906 P4= 15.9737
V6= 0.431702 V7= 0.121391 V8= 0.242782 V9= 1.38971
F= 10
J= 10 P1= 3.78265 P2= 1.18467 P3= 2.36934 P4= 5.30211
V6= 0.760793 V7= 0.224457 V8= 0.448915 V9= 1.851
F= 10
J= 15 P1= 3.05189 P2= 1.11127 P3= 2.22253 P4= 2.49934
V6= 1.02631 V7= 0.321138 V8= 0.642275 V9= 2.06844
F= 10
J= 20 P1= 2.55595 P2= 1.07555 P3= 2.1511 P4= 1.39464
V6= 1.24867 V7= 0.41471 V8= 0.82942 V9= 2.18977
F= 10
J= 25 P1= 2.19829 P2= 1.05514 P3= 2.11029 P4= 0.862492
V6= 1.43993 V7= 0.506508 V8= 1.01302 V9= 2.26481
F= 10
J= 30 P1= 1.92894 P2= 1.04229 P3= 2.08458 P4= 0.572233
V6= 1.60774 V7= 0.597187 V8= 1.19437 V9= 2.31459
F= 10
J= 35 P1= 1.71946 P2= 1.03364 P3= 2.06727 P4= 0.399698
V6= 1.75734 V7= 0.687113 V8= 1.37423 V9= 2.34937
F= 10
J= 40 P1= 1.55243 P2= 1.02752 P3= 2.05503 P4= 0.290405
V6= 1.8924 V7= 0.776507 V8= 1.55301 V9= 2.37463
F= 10
J= 45 P1= 1.41664 P2= 1.02302 P3= 2.04605 P4= 0.21768
V6= 2.01564 V7= 0.86551 V8= 1.73102 V9= 2.39357
F= 10
J= 50 P1= 1.3045 P2= 1.01963 P3= 2.03926 P4= 0.167342
V6= 2.12914 V7= 0.954818 V8= 1.90844 V9= 2.40813
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